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-ABSTRACT-

Elastic Behavior and Stress Evaluation in the Tooth Root Fillet Area of a Helical Cylindrical Gear

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Introduction

Helical gears represent an essential category of components in modern mechanical engineering, being preferred in most power transmission systems, ranging from heavy industry to automotive and aerospace applications. This preference is justified by their significant operational advantages: they ensure gradual and continuous meshing, which minimizes shock, substantially reduces operating noise, and allows the transmission of much higher torques compared to spur gears. However, their geometric complexity also introduces major challenges, particularly with regard to stress evaluation and fatigue durability assurance.

The critical point of these components, which most frequently dictates fatigue failure, is the tooth fillet region. Here, the abrupt change in cross-sectional area, combined with the complex forces generated by helical meshing (including bending, axial forces, and a non-uniform load distribution along the tooth width), produces maximum stress concentrations. An inaccurate evaluation of these critical stresses directly leads either to inefficient over-dimensioning or, far more dangerously, to under-dimensioning, which results in premature mechanical failures. Conventional design methods, based on internationally recognized standards such as ISO 6336 or AGMA Standard 2001-D04, although fundamental, rely on empirical correction factors that often prove insufficient or inaccurate in capturing local details—such as the precise influence of tool geometry (e.g., protuberance fillet) or residual stresses.

Legend

No,	Abbrev.	Significance
01	b	Face width of gear
02	d	Pitch diameter
03	d_a	Addendum (tip) diameter
04	d_b	Base diameter
05	d_f	Base diameter
06	d_m	Root diameter
07	d_1	Mean bearing diameter
08	d_2	Pitch diameter of the pinion
09	ε	Pitch diameter of the gear wheel
10	*F*	Contact ratio

11	F_{fZ}	Force
12	F_a	Tangential force at the base circle
13	F_{bt}	Axial load
14	F_r	Tangential force at the base circle
15	h	Radial load
16	h^*_{0a}	Tooth height
17	h^*_{0f}	Reference addendum coefficient
18	h_a	Reference dedendum coefficient
19	h_f	Addendum height
20	m_n	Dedendum height
21	m_t	Normal module
22	p_E	Transverse module
23	p_n	Helix pitch
24	p_t	Normal pitch
25	WL	Transverse pitch
26	x	Specific load
27	z	Profile shift coefficient
28	α_n	Number of teeth
29	α_t	Normal pressure angle at pitch cylinder
30	β	Transverse pressure angle at pitch cylinder
31	T	Helix angle at pitch cylinder
32	K_{FX}	Torque
33	S_F	Rotational speed
34	σ_F	Tangential force
35	ν	Radial stress

36	E	Virtual number of teeth
37	u	Young's modulus
38	β_b	Axial force
39	Z_H	Form factor
40	Z_M	Load distribution factor
41	Z_ϵ	Helix angle factor
42	Z_β	Peripheral speed factor
43	K_{HL}	Internal dynamic factor
44	Z_L	Overload factor
45	Z_R	Life factor
46	Z_V	Load distribution factor along face width for root stresses
47	Z_W	Safety factor for root fatigue
48	K_{HX}	Tooth bending stress
49	$K_{H\beta}$	Poisson's ratio
50	S_H	Longitudinal elasticity modulus
51	σ_H	Transmission ratio
52	n	Helix angle at base diameter
53	F_t	Rolling point factor
54	F_r	Material factor
55	Z_{v1}	Contact ratio factor
56	X	Helix inclination factor
57	Fa	Life factor
58	Y_F	Lubrication factor
59	Y_ϵ	Surface quality factor
60	Y_β	Sliding velocity factor

61	v_1	Flank hardness ratio factor
62	K_v	Size factor
63	K_0	Load distribution factor on tooth flank
64	K_L	Hertzian stress safety factor
65	W_T	Contact stress
66	K_0	Transmitted force
67	Q_v	Overload factor
68	K_v	Precision grade
69	C_{mc}	Dynamic factor
70	C_{pf}	Load correction factor
71	C_{pm}	Pinion proportion factor
72	C_{ma}	Profile modification factor
73	Ce	Alignment factor (discretization)
74	KH	Mesh alignment correction factor
75	KB	Load distribution factor
76	YJ	Tooth thickness factor
77	σ_b	Tooth root stress form factor
78	ZE	Root stress
79	ZI	Elasticity coefficient
80	ZR	Surface factor
81	σ_c	Surface factor
82	FEM	Contact stress in gear mesh
83	MEF	Finite Element Method
84	SOLID186	Finite Element Method (Romanian version)
85	CONTA174	20-node 3D structural element

86	TARGE170	Contact element for surface interaction
87	D1–D6	Target element in ANSYS for contact coupling
88	P1–P6	Stress measurement points in tooth fillet region
89	Dorn	Force application positions via mandrels, equidistant
90	C++	Experimental component used for point load application
91	σ_{VM}	Programming language used for overlay code
92	Δσ	von Mises equivalent stress (FEM analysis)
93	ε_VM	Stress variation between load positions
94	Pinball Region	von Mises equivalent strain
95	Augmented Lagrange	ANSYS parameter for defining virtual contact zone
96	$F_{aplicată}$	Contact formulation method in ANSYS
97	σ_{rac}	Applied force via test stand (per mandrel)

Chapter 1. Current State of Experimental and Analytical Research on Helical Cylindrical Gears

Mechanisms for transmitting and transforming rotational motion represent an essential element in the development of mechanical engineering, with gears occupying a central position in this context. Throughout history, these mechanisms have evolved from primitive wooden forms to today's sophisticated high-precision systems employed in numerous technical fields.

The earliest uses of gears date back to antiquity, within the Egyptian, Greek, and Roman civilizations, where they were employed in locking systems, gates, and water extraction mechanisms. A major milestone in the history of these mechanisms is represented by the Antikythera Mechanism (2nd century BC), an extremely complex construction for its time, capable of performing astronomical calculations. It is considered the first analog computer and demonstrates the advanced level of understanding of mechanics and metalworking achieved in Ancient Greece.

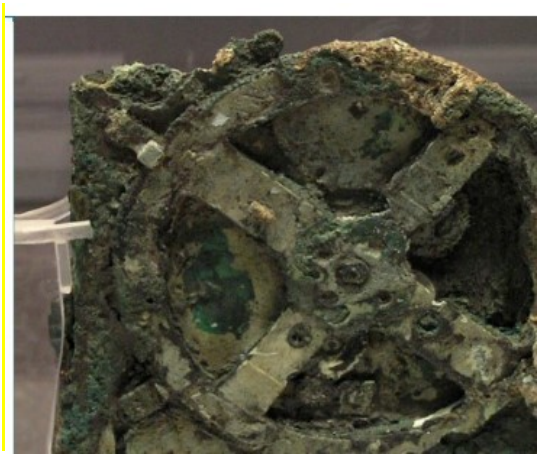


Fig. 1.2. The large fragment of the Antikythera Mechanism [5]

Archimedes, renowned for his numerous contributions to science and engineering, was the first to describe the worm gear mechanism, an important technical solution for transmitting motion between perpendicular shafts.

As mathematics developed, the form of gear teeth was progressively refined. Initially, tooth profiles were established empirically; however, they were later defined mathematically (e.g., involute, cycloidal), ensuring constant transmission of motion and smooth operation. These advances became increasingly significant from the end of the Renaissance onward.

During the Industrial Revolution (18th century), the demand for efficient transmission of high forces led to the widespread development of gear systems. James Watt, in an effort to circumvent a patent, designed a planetary gear system for his steam engines, which proved both effective and innovative. Subsequently, the rise of the automobile industry

generated a new wave of innovations: ÉmileLevassor and Louis Renault laid the foundations of the modern transmission, while Wilhelm Maybach improved the system through the introduction of sliding-mesh gears and helical gearing, which provided quieter operation and greater load capacity.

At the same time, in the field of machine tools, gears became key components for mechanical precision. The introduction of change gears by Henry Maudslay enabled threading with high accuracy. Although gear failures were common during the First World War, they were gradually replaced by more reliable systems, owing to their advantages: precision, durability, safety, and the ability to achieve variable speeds.

In the modern era, the analysis and design of gears has been revolutionized by the advent of computers and numerical methods. Classical analytical models (Hertz, Lewis, Timoshenko) have been complemented by the Finite Element Method (FEM), which allows detailed analysis of stress distribution, deformations, and the actual behavior during operation. Researchers such as Wilcox, Vijayakar, Chabert, and Tsay have contributed to the development of these models, analyzing tooth contact, the effects of assembly errors, as well as wear and material fatigue.

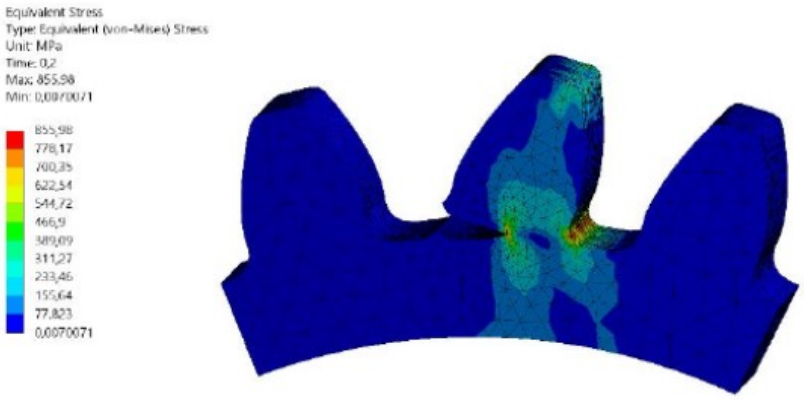


Fig. 1.3. Finite Element Simulation of Gears [22]

Test Rigs for Tooth Root Stress under Static Load

Moreover, modern gear design relies on specialized software (e.g., CATIA, SolidWorks, Inventor, Ansys), which enables comprehensive simulation of gear operation under real conditions. These tools assist in optimizing flank profiles, selecting appropriate materials, and evaluating resistance to scuffing, wear, and other forms of degradation, thereby enhancing transmission reliability and durability.

In conclusion, gears have evolved from rudimentary tools to high-precision components that are indispensable in modern mechanisms. This evolution reflects not only technical progress but also the continuous adaptability of mechanical solutions to the increasingly complex demands of modern society.

The analysis of tooth root stress under static loading is essential for understanding the initial behavior of gears under load, providing a solid foundation for design and optimization. In this context, experimental methods and numerical models play a crucial role in assessing stress distribution and validating theoretical assumptions.

Among the most commonly used experimental techniques are strain gauges—precise instruments for measuring local deformations—which enable the determination of stresses in the tooth root region, where the highest stress concentrations occur. These devices operate based on the variation of electrical resistance of a sensitive element and are integrated into Wheatstone bridge circuits to detect minute changes. Applications may include telemetry systems or slip rings for signal transmission, even under static conditions.



Fig. 1.4. Pre-wired Strain Gauges [28]

From an analytical and numerical perspective, 2D and 3D modeling of gears—particularly helical gears—allows realistic simulation of tooth behavior under load. Methods such as Finite Element Analysis (FEA) provide detailed results regarding stress distribution, the influence of geometry, and contact conditions, facilitating the identification of crack initiation zones and the optimization of gear design.

International standards (ISO 6336, DIN, etc.) employ models based on tooth compliance—an essential parameter for evaluating deformability and load distribution. Unlike simple models that consider only tooth stiffness, modern approaches also account for the flexibility of shafts, hubs, and bearings, taking into consideration phenomena such as Hertzian deformation, bending, shear, and axial compression.

Numerous studies, ranging from classical research by Weber and Walker to modern Finite Element Method (FEM) approaches, have addressed the analysis of stresses, compliance, and crack propagation in the tooth root region. Experimental and theoretical investigations have demonstrated that contact stiffness is significantly influenced by tooth geometry, load position, material properties, lubrication conditions, and the presence of defects such as cracks.

Recent research has also extended to nonlinear behavior, transmission dynamics, the influence of profile errors, non-uniform load distribution, and temporal variations in

contact stiffness. The effects of cracks on stiffness variation have been studied, along with early fault prediction methods using analytical, experimental, or hybrid techniques.

In conclusion, the analysis of tooth root stress represents a complex, multidisciplinary topic that combines high-precision experimental methods with advanced numerical modeling. The integration of these approaches contributes to improving the reliability, durability, and performance of gears, particularly in critical applications.

Helical cylindrical gears provide efficient and quiet torque transmission, making them the preferred choice for demanding applications due to the progressive tooth engagement. They ensure reduced wear, high reliability, and superior performance even at high speeds. Although their manufacturing is complex, advances in CAD, FEM, and dynamic simulation enable their optimization. Current trends focus on customizing tooth profiles and integrating modern technologies, maintaining the relevance of these gears in the face of emerging alternatives.

The proposed research and development activity aims to establish an integrated methodology for analyzing the mechanical behavior of helical cylindrical gears, particularly for carburized gears. The study focuses on comparing analytical methods according to ISO and AGMA standards, developing an accurate geometric tooth model, performing numerical FEM simulations, and experimental validation using strain gauges. The objective is to achieve precise evaluation of stress distribution and to enhance the durability of gears under real operating conditions.

Chapter 2. Calculation Approach in ISO 6336 and AGMA Standard 2001–D04

Chapter 4 addresses the evaluation of mechanical stresses in helical cylindrical gears, with a focus on tooth root and contact stresses. The analytical methods provided by ISO 6336 and AGMA 2001–D04 standards are comparatively analyzed. Although similar in structure, these methods exhibit significant differences regarding the considered parameters, calculation assumptions, and the results obtained.

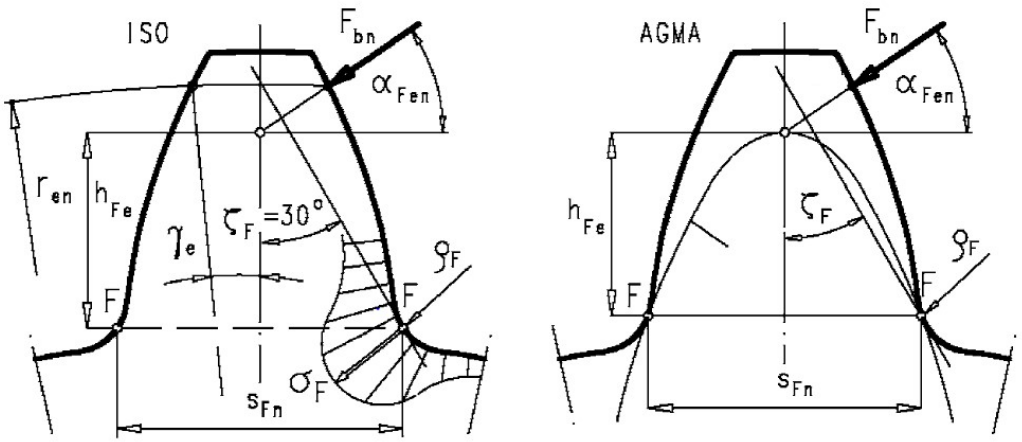


Fig. 4.1. ISO and AGMA Load Schemes [64][66]

To obtain a more accurate estimate of the real behavior under load, the Finite Element Method (FEM) is proposed, allowing the analysis of gears with corrected geometries that cannot be rigorously addressed using classical methods. The Hertzian model is employed as the basis for contact stress analysis, adapted to the complexity of gear geometry.

The importance of detailed tooth contact modeling and the influence of dynamic geometry on stress distribution are emphasized. Additionally, the limitations of analytical methods in comparison with numerical simulations are highlighted, and a rigorous comparative analysis (ISO vs. AGMA vs. FEM) is proposed under the same operating conditions and geometry.

In Subchapter 4.3, the calculation formulas according to the standards will be applied using a defined set of geometric and functional data for the analyzed gear.

Name	Symbol	Unit	Z1	Z2
Number of teeth	z	-	16	16
Normal module	m_n	mm	6	
Profile shift coefficient	x	-	0.682	0.625
Face width	B	mm	150	
Normal pressure angle	A	°	20	
Helix angle at base diameter	B	°	10	
Transverse pressure angle	α_t	°	20.2836	
Operating pressure angle	α_w	°	28.4547	
Center distance	A12	mm	104	
Transverse module	m_t	mm	6.0926	
Pitch diameter	d	mm	97.4810	97.4810
Rolling diameter	d_w	mm	104	104
Base diameter	d_b	mm	91.436	91.436
Addendum height	h_a	mm	10.092	9.75056
Tooth dedendum coefficient	K	-	0	
Tooth root clearance	c	mm	1	
Dedendum height	h_f	mm	3.408	3.74944
Outside diameter	d_a	mm	117.665	116.982
Root diameter	d_f	mm	90.665	89.9821
Face contact ratio	ϵ_α	-	1.33459	
Helix contact ratio	ϵ_β	-	1.38185	

Chapter 3. Advanced 2D Modeling of Involute Gears – Simulation and Validation

In the context of modern engineering, 3D modeling of gears and validation of their behavior through advanced numerical simulations represent essential steps in the design and optimization of transmission systems. Due to the geometric complexity and the loads to which gears are subjected in operation, a faithful virtual model is required to

accurately reproduce both the shape and mechanical behavior of the components. In this regard, 3D modeling was performed using Siemens NX 12, a high-performance software widely used in the machine-building industry, due to its advanced capabilities for parametric modeling, simulation, and integration with Finite Element Method (FEM) analyses.

The modeling of the pinion began with the definition of the blank, a cylindrical body with an outer diameter of 117.665 mm and a length of 150 mm, created through an extrusion operation. This body served as the base on which the entire tooth geometry was subsequently constructed. To generate the teeth, the process started from the transverse profile of a single tooth, obtained from a set of coordinates that precisely define the involute shape. This profile was then used in a "swept" operation, which allowed the creation of the three-dimensional shape of the space between two teeth, accurately respecting the functional geometry of the engaged teeth.

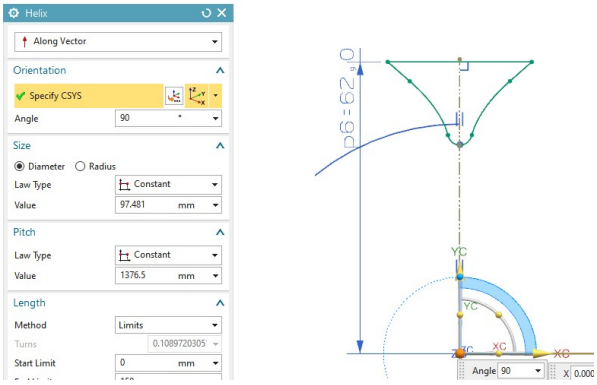


Fig. 5.4. Tooth Helix Generation

After obtaining a single tooth space, it was duplicated using the "circular pattern" function, which generated the entire pinion tooth system, respecting the circular pitch and symmetry specific to an external spur gear. This approach not only enabled efficient and precise modeling of the pinion but also allowed easy modification of geometric parameters in case further optimizations were performed during the analysis phase. Additionally, the resulting geometry was fully parametric, providing maximum flexibility for adapting the design to real operating conditions or the requirements of the specific application.

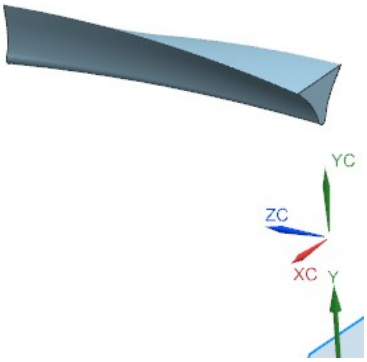


Fig. 5.5. Tooth Negative

Once the 3D model of the pinion was completed, it was imported into a simulation environment for Finite Element Method (FEM) analyses. The purpose of these simulations was to evaluate the mechanical behavior of the pinion under real operating conditions, particularly regarding stress distribution in the tooth contact area, the risk of fatigue initiation, and behavior under dynamic loads. By applying a concentrated load on the flank of a single tooth, it was possible to analyze how the force is transmitted to adjacent teeth and to identify critical areas with an increased risk of cracking or plastic deformation.

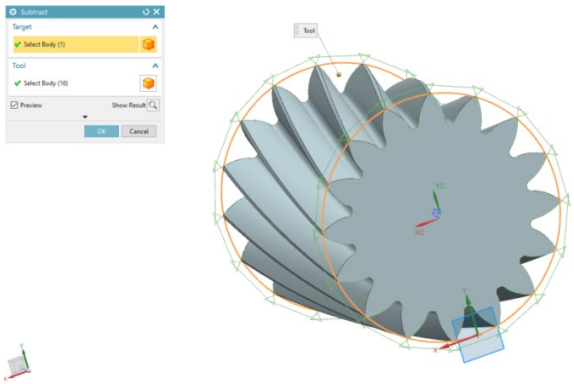


Fig. 5.7. Generated Pinion

The results obtained from the numerical simulations were essential for validating the design and identifying the need for potential optimizations. Based on the stress distribution and the material behavior under cyclic loading, adjustments to the tooth profile were proposed, particularly in the fillet region between the tooth root and the flank, where stress concentrations frequently occur. Additionally, the possibility of applying local heat treatments, such as carburizing or induction hardening, was considered to increase the hardness of the contact surfaces and improve wear resistance.

In conclusion, the integration of 3D modeling processes with Finite Element Method (FEM) simulations provides a comprehensive understanding of the pinion's behavior under real operating conditions. This approach allows not only the design of a functional gear but also its optimization in terms of reliability, durability, and mechanical performance. Therefore, the use of these advanced tools becomes indispensable in modern gear design, significantly contributing to the reduction of production costs, minimization of development time, and enhancement of the final product's competitiveness.

Chapter 4. Evaluation of Pinion Geometric Accuracy: Comparative Study Between Code-Generated Models and NX12 Library Models

In this section of the thesis, a detailed comparative analysis of gear geometry is presented, comparing two methods for generating the involute profiles of pinions: one developed using a proprietary algorithm for parametric modeling within the NX12 design environment, and the other based on the standard library provided by the same software. The main objective of the study is to evaluate the geometric accuracy of each method and to determine their suitability for various stages of the design and structural analysis process of gears. Particular emphasis is placed on the accuracy of the tooth involute profile, as well as the critical fillet region of the tooth.

Methodology for Verifying Involute Profile Accuracy

To evaluate the geometric differences between the generated models, a rigorous methodology based on the fundamental principles of involute profile generation theory was adopted. The process began with modeling the pinion's base circle, a key element for defining the involute tooth profile. Subsequently, two-dimensional projections of the tooth profile were extracted on the frontal plane to allow precise measurements of relevant dimensions. These measurements were taken at five different angles (10°, 16°, 22°, 28°, 35°), strategically selected to represent significant portions of the tooth flank, from the root to the tip.

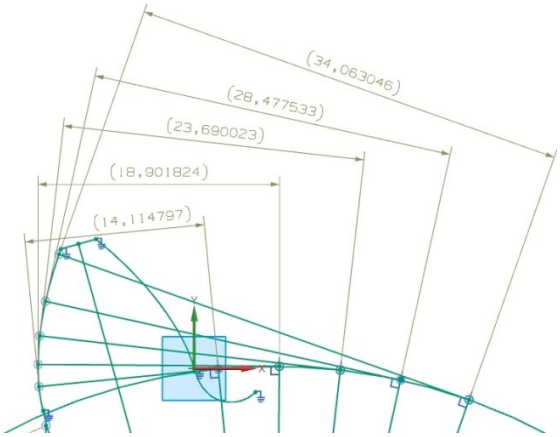


Fig. 6.4. Detail of Generated Gear

From the intersection points of these angular lines with the base circle, tangents were drawn in the direction of the involute, extending them until they intersected the actual contour of the tooth profile. This geometric approach allowed the definition of the involute thickness at different depths, measured relative to the base circle, and facilitated the precise comparison of tangent segment lengths between the algorithmically generated models and those from the NX12 library. The results highlighted minor differences between the two methods, which were significant only in certain critical regions of the profile.

Methodology for Verifying Fillet Accuracy

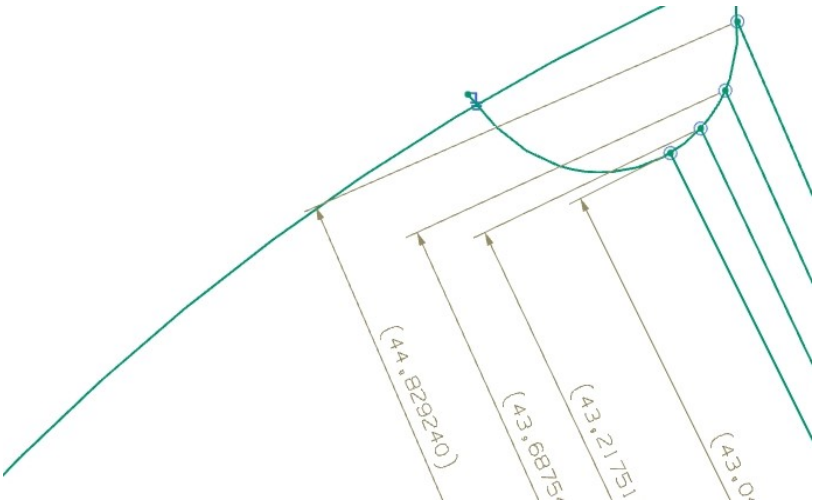


Fig. 6.5. Fillet of Generated Gear

In addition to the involute analysis, particular attention was given to the tooth fillet region, which is critical for the mechanical strength of gears. This area was evaluated through measurements of the radius of curvature and distances between relevant points, for both the model generated by the proprietary code and the one from the NX12 standard library. Comparing these parameters provided insights into the fidelity of the fillet geometry and potential differences that could influence the mechanical behavior of the gear, especially regarding wear resistance and performance under load.

Results and Analysis

Table 6.1. Comparison of Tangent Segment Lengths (Code-Generated Model vs. NX Library)

Profile Angle (degrees)	Tangent Segment Length – Code Model (mm)	Tangent Segment Length – Library (mm)	Difference (mm)
10	14.555156	14.59809	-0.042934

16	19.342752	19.385596	-0.042844
22	24.130351	24.172272	-0.041921
28	28.917910	28.960854	-0.042944
35	34.454674	34.546295	-0.091621

To verify the involute profile, six control points were selected along the involute curve. Each point was characterized by its polar coordinates, and the rolling parameter was calculated for each point. These calculations were essential for determining the point's position along the involute trajectory and for comparing the theoretical angles with the actual ones. Additionally, mathematical formulas were implemented in Excel to provide an extra verification of angular deviations.

The results showed that, compared to the model generated from the NX12 library, the model created using the proprietary algorithm exhibited a slightly higher angular deviation, particularly in the tooth tip regions. However, this deviation is considered acceptable within the context of the project objectives, given that the proprietary model provides superior control over construction parameters and allows for a more precise adaptation of the profile to the actual tooth geometry. Consequently, the algorithmically generated model demonstrates higher accuracy in the critical regions of the profile, which is essential for the long-term performance of the gear.

Conclusions

Code-generated profile:

rb	s	r	tetagrd	teta rad	fi	fi rad	tetatheoretical	tetagrd	eroare (grd)
45.7	14.555	47.961	0.6427	0.0112	18.309	0.3195	0.01025	0.5876	0.055133
45.7	19.342	49.624	1.3683	0.0238	24.309	0.4242	0.02301	1.3188	0.049474
45.7	24.130	51.679	2.4742	0.0431	30.309	0.5289	0.04242	2.4305	0.043748
45.7	28.917	54.080	3.9843	0.0695	36.309	0.6337	0.06887	3.9461	0.038279
45.7	34.454	57.232	6.2563	0.1091	43.309	0.7558	0.10862	6.2239	0.032391

Following the detailed analysis, it can be concluded that the code-generated profile method is preferable to using the NX12 library, particularly due to the greater control it provides over the geometric parameters of the profile and fillet. Although the library model exhibits a smaller error regarding the involute geometry, the algorithmically generated model allows for a more accurate adaptation to the specific requirements of the

project. This ensures better compatibility with the mechanical performance and reliability demands of the gears, having a significant impact on subsequent stages of numerical analysis and comparison with experimental data.

Thus, the algorithmic method proves to be more robust and efficient, offering greater potential for further optimizations, particularly in the context of achieving specifically customized geometries for complex industrial applications.

Chapter 5. Test Rig

In this research, the main objective was to perform a comprehensive analysis of the mechanical behavior of helical cylindrical gears with significant axial widths, within an experimental framework that faithfully reproduces real operating conditions. The previous theoretical and numerical approach examined the stress behavior of the gears using both analytical models and Finite Element Method (FEM) simulations. However, the numerical models were limited by simplifying assumptions (e.g., isotropic material, idealized contact, perfectly controlled loading conditions) that failed to fully capture the complexity of the actual behavior of gear teeth. Therefore, to complement and validate the results obtained through numerical methods, the study included an experimental phase conducted on an innovative test rig.

Role of the Experimental Phase

The experimental rig was designed to enable precise measurement of stress and strain distribution on the tooth flanks and fillet, replicating the real operating conditions of a loaded gear. The experimental phase was essential for identifying critical stress regions as well as the specific characteristics of load transmission that cannot be observed through simulations. Through this approach, the aim was to complement the numerical results with concrete experimental data, allowing a direct validation of the actual behavior of the teeth under various operating conditions.

Design and Construction of the Experimental Rig

A fundamental aspect of the research was the design and construction of a dedicated experimental rig that allows the evaluation of the mechanical behavior of gears with complex geometries, characterized by helical cylindrical teeth and significant axial widths. The experimental rig was integrated into a universal testing machine, which ensures the application of controlled compressive forces while maintaining stable testing conditions. The load application mechanism uses a self-locking screw–nut system, manually operated, which acts on a longitudinal carriage, mounting a specially designed probe for the precise application of tangential force to the tooth.

To ensure measurement accuracy, the system includes a spring mechanism for the automatic retraction of the probe during adjustment phases, thus preventing unwanted contact with the measuring equipment. Additionally, the exact position of the force application point is determined using a linear measurement system (graduated ruler and fixed indicator needle), which allows direct reading of the probe displacement and its correlation with the contact position on the tooth surface.

Instrumentation and Experimental Data Collection

The instrumentation was designed to enable the comparison of the behavior of the active and passive flanks of the tooth, using strain gauges strategically placed on each flank. These gauges allowed the identification of tensile and compressive states generated by the bending moment induced by the tangential component of the applied force. The

measurement system was designed to acquire strain (ε) values at each strain gauge using a Wheatstone bridge configuration, and the analog signals were digitized and transmitted in real time to a computing unit for processing and graphical representation of the results (force–deformation curves).

To ensure high repeatability, ten independent test sets were conducted for each combination of applied force and force application point on the tooth surface. This approach allowed the calculation of robust statistical averages and narrow confidence intervals, reducing the influence of random variations on the experimental data.

Moreover, prior to the actual testing, the rig underwent a calibration and validation phase, which included verifying the linearity of the measurement system response, testing repeatability under constant loads, and comparing experimental results with those obtained from numerical simulations for a reference case. This process ensured the accuracy of the measurements and reinforced the reliability of the data obtained from the experiments.

Validation of Results and Contribution of the Experimental Rig

The results obtained from the experimental tests were directly relevant for the design, validation, and optimization of gears with complex geometry. The developed experimental rig enabled a detailed evaluation of stress and strain distribution across the entire tooth surface and in the fillet region, thereby highlighting behaviors that are not fully captured by numerical models.

Furthermore, the experimental infrastructure provided a versatile and modular framework capable of testing a wide range of gear configurations, with applicability in both academic research and industrial applications. This innovative experimental rig represents a significant contribution to the field of tribology and gear mechanics, with the potential to optimize design solutions and improve the reliability of mechanical transmissions. In addition, by enabling the application of progressively concentrated forces and maintaining loads within the elastic range of the material, the rig allows a detailed assessment of tooth behavior under conditions closely resembling real operational scenarios.

Impact of the Research

The research makes a significant contribution to the detailed understanding of the real behavior of gears, especially under extreme operating conditions, and has an important impact on the optimization of design solutions. Through the use of the experimental rig, valuable experimental data are obtained that enable the validation and adjustment of existing numerical models, playing a crucial role in enhancing the performance and reliability of modern mechanical transmissions.

In conclusion, the innovative experimental rig developed within this research represents a key link in understanding the mechanical behavior of gears with complex geometries,

providing a valuable research tool for mechanical engineering and contributing to the advancement of knowledge in the field.

The 6 mm module was chosen as the optimal size for the analyzed tooth, considering the necessity of allowing proper installation of strain gauges. The tooth fillet area is a sensitive point where fatigue cracks frequently occur, and a smaller module would have reduced the space available for reliable sensor placement. This choice is essential to ensure the effective implementation of the measurement system and to obtain precise data on the tooth's behavior under load.

The 150 mm width of the gear was chosen to provide adequate support for the placement of multiple strain gauges across the entire tooth surface. This decision is crucial because the interaction between teeth in a helical cylindrical gear develops progressively along the full width, and only a sufficiently large width allows a complete study of the stress distribution across the entire contact area. Therefore, accurate measurement can be achieved only if the sensors are properly positioned along the full length of the tooth.

The gear's 16 teeth introduce a phenomenon known as undercutting, where material in the fillet region is removed more than is ideal. This can weaken the tooth structure in this area, making it more susceptible to stress concentrations and the formation of fatigue cracks. Although undercutting may affect the strength of the tooth root, it was intentionally selected to analyze the material behavior under varying loading conditions.

The materials selected for the manufacture of the gear and pinion represent a compromise between mechanical performance and cost. The pinion is made of a case-hardening steel, providing a hard surface capable of withstanding continuous wear and fatigue loads. In contrast, the gear is made from a more easily machinable and economical material, while still maintaining sufficient strength and toughness to handle the typical loads encountered in standard applications.

A crowning machining process was chosen to modify the tooth profile, which helps control stress concentration. This approach facilitates the measurement of tooth behavior even under low loads, allowing early detection of fatigue cracks. Consequently, it is possible to monitor with precision the tooth root area, where the most critical stress accumulations occur.

The strain gauges were strategically positioned on the tooth surface to analyze the stress distribution under load. These measurements are essential for understanding how the material responds to applied forces. The data acquisition system was carefully calibrated to ensure precise and reliable measurements, and the signals obtained from the strain gauges are recorded and analyzed in detail.

The experimental setup utilizes a testing rig in which the pinion is fixed to prevent any unwanted rotational movement, while the strain gauges are connected to a Wheatstone bridge, a highly precise device that allows the measurement of resistance variations. These signals are then processed to obtain information regarding the material behavior under stress. A universal testing machine is used to apply controlled loads, and the data

acquisition system is integrated with software that records and analyzes the results in real time.

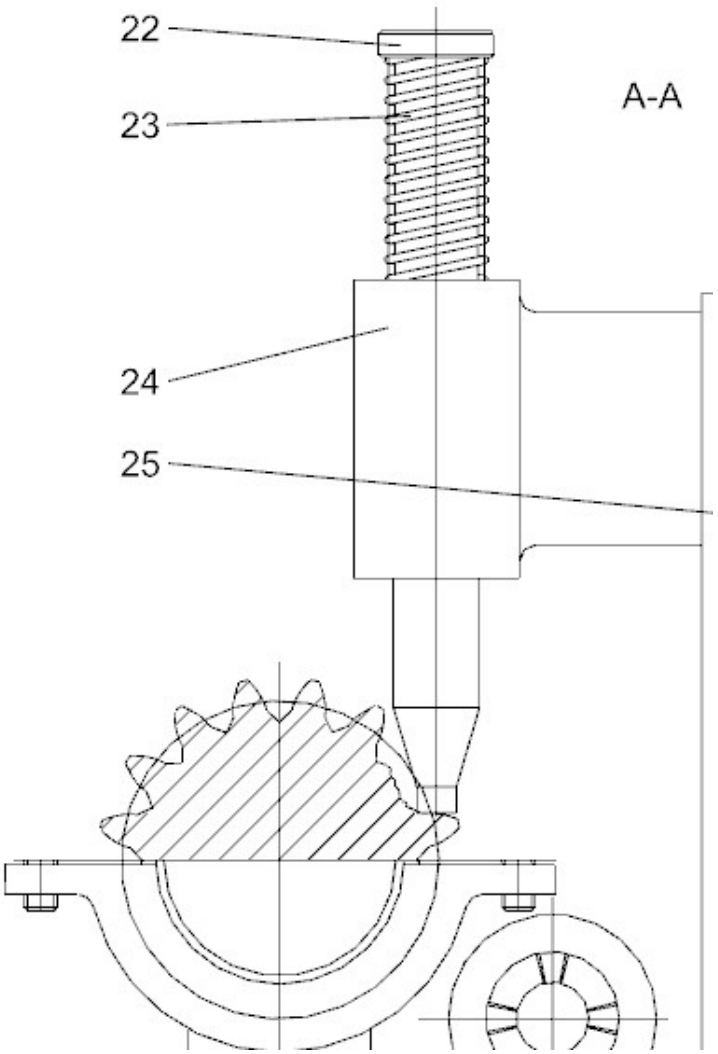


Fig. 7.2. Testing rig sketch (side view)

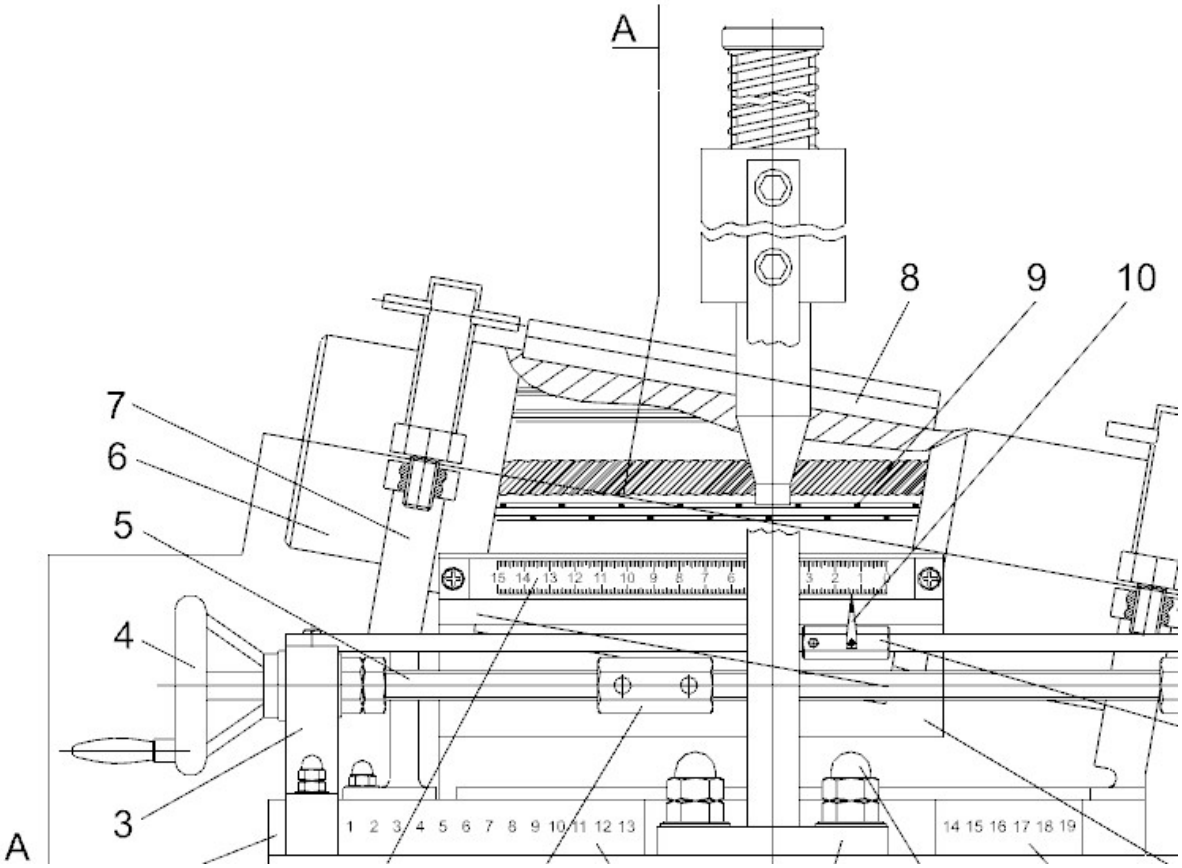


Fig. 7.3. Testing rig sketch (front view)

To validate the accuracy of the data, the tests are repeated multiple times, and statistical analyses are performed to identify and eliminate outlier values that could compromise the integrity of the conclusions. The obtained data are processed using polynomial interpolation methods, which allow the construction of continuous load–stress functions. These functions are then used to estimate the stresses at any given load level.

To prevent the influence of outlier values on the analysis, the Grubbs test is employed to detect extreme values within the dataset. This method ensures that only valid measurements are included in the final analysis, providing results that are accurate and representative of the material's behavior under load.

The research was conducted in a controlled environment using an experimental rig, where 10 datasets were collected to analyze the elastic behavior of the tooth under point load. During the tests, all equipment and sensors were calibrated, and parameters were carefully monitored to ensure repeatability of the results. The Grubbs test was applied to identify and remove outliers from the datasets, thereby improving the accuracy of the analysis.

The investigations conducted on the gear tooth revealed an elastic behavior, but with a non-uniform and asymmetric distribution of deformations across the tooth width,

significantly influenced by the positioning of the probe through which the force is applied on the tooth flank. All analyzed deformation curves exhibited an almost linear evolution of deformations with respect to load, suggesting that the material remains within the elastic domain throughout the entire loading range.

Minor deviations in slope observed at higher loads were insufficient to indicate material plasticization and are instead associated with complex elastic effects, specific to the behavior of an assembly under eccentric loading, such as:

- Local stiffening of the contact area under the probe;
- Interaction between axial bending and torsion along the width direction, a phenomenon typical for helical gears.

Distribution of Deformations Across the Tooth Width:

One of the most important aspects of the research was the non-uniform distribution of deformations across the entire tooth width. Specifically:

- **Gauge 1**, located near the point of load application, recorded the highest deformations. This can be explained by the concentration of the load near the application point, where stresses are maximal.
- **Gauges 5 and 6**, situated farther from the load application point, exhibited significantly lower deformations, suggesting a gradual dissipation of stresses as the distance from the direct contact area with the probe increases. This phenomenon reflects localized elastic behavior, characteristic of the "edge loading" effect.
- In the **transition zone** between the regions of high stress concentration and the less stressed areas (around Gauge 4), deformations were intermediate, indicating a gradual redistribution of stresses from the central zone toward the edges.

This asymmetry in the deformation distribution is an essential characteristic of point loading, particularly in helical gears, where the combined effects of torsion and bending cause stress concentrations at the tooth extremities, even when the load is applied centrally.

Pin Positioning and Its Impact on Deformations:

Positioning the pin close to the tooth root had a significant impact on the tooth’s behavior under load. An important aspect is that this resulted in a shorter lever arm, reducing the bending effect of the tooth. Consequently, the torsion generated by the applied load was considerably diminished, and the deformation distribution became more localized, leading to higher stress concentration near the point of application (Strain Gauges 1 and 2).

Edge Loading and Local Stiffening:

As the distance from the load application point increases (towards Strain Gauges 5 and 6), deformations decrease significantly, indicating a load dissipation effect characteristic of "edge loading." In this context, the influence of the applied load on the tooth diminishes, suggesting that only the tooth extremities are sensitive to localized point loads. Additionally, the regions at the tooth edges (Strain Gauges 1 and 6) exhibited local stiffening, which may reduce the risk of permanent deformation in these areas. At the same time, this indicates that the tooth edges are susceptible to surface defects, such as pitting or micro-cracks, when loads are concentrated in these zones.

Structural Behavior Under High Loads:

At high loads (above 40–60 kN), the deformation curves became steeper, indicating nonlinear behavior and a reduction in local stiffness. This phenomenon is indicative of nonlinear contact effects, where deformations no longer follow a linear trend, and the material begins to respond in a more complex manner under elevated loads. Such conditions can lead to the onset of plastic deformations or, in the worst case, permanent damage in regions of stress concentration.

Numerical Simulations and Experimental Validation:

To validate the experimental results, numerical simulations were performed using the superposition of effects method to replicate a force distribution as close as possible to the actual gear conditions. The simulation results were compared with the experimental data, and the similarities between the two approaches were evident. These simulations proved useful for providing a more detailed understanding of the tooth behavior under high loads, as well as for identifying critical regions in terms of material fatigue and potential defect risks.

General Conclusions and Design Recommendations:

1. The asymmetric distribution of deformations indicates the need to apply geometric corrections, such as crowning or tooth profile modifications, to uniformize the load across the entire tooth width and reduce the risk of structural defects.
2. • Positioning the indenter close to the tooth root proved beneficial for reducing bending and torsion, but it also highlighted the importance of a balanced tooth design to prevent load concentration at the edges.
3. • Torsional and bending effects should not be underestimated during the design process, as they can lead to localized stress concentrations in specific tooth regions, which are most susceptible to defects such as pitting or microcracks.
4. • It is recommended that point loads be distributed as uniformly as possible across the full tooth width to maximize the durability and performance of the gear assembly.

Chapter 6. Simulation of the Mechanical Behavior of Cylindrical Gears: Tooth-by-Tooth Engagement Model

Summary of the Real Engagement Simulation

The simulation of real gear engagement aims to replicate as accurately as possible the operating conditions of the gears. To achieve this, it is essential to provide a detailed description of the geometry and interaction between the mating gears. The process begins with the rigorous definition of the shape and dimensions of the gears, which forms the basis for all calculations related to contact, stress distribution, and motion transmission.

Geometric Model

The geometric model of the gear was constructed based on the pinion used in the experimental tests and previous simulations with point-loaded probes. The model geometry precisely replicates the shape of the actual component, with a pinion sector used to optimize computational resources. The sector includes the active contact area between the teeth, while regions not engaged in meshing were omitted from the model, as they do not influence the local stress distribution.

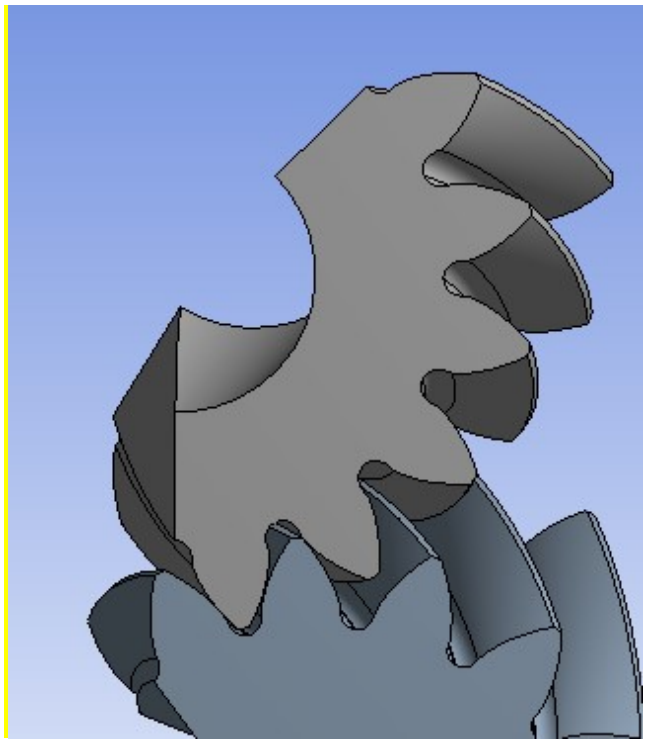


Fig. 8.1. Gear Sector

The model was created in DesignModeler and imported into ANSYS Workbench for 3D static simulation. This approach allows rapid modifications of the geometry and ensures comparability between numerical and experimental results. The analysis focused on

evaluating the stress distribution in the tooth root fillet area, which is critical for the fatigue behavior of the gear.

Materials

The material used for the simulation is 18CrNiMo7-6, an alloy steel with excellent strength and durability properties, commonly employed in power transmission applications. The mechanical properties applied in the simulation are an elastic modulus of **2.1×10^5 MPa**and a Poisson's ratio of 0.28, which reflect the elastic behavior of the material under load. These characteristics are essential for ensuring wear resistance and fatigue strength, which are critical factors in evaluating the durability of the gear assembly.

Contact

For the type of contact, a **Frictionless** assumption was selected, meaning that tangential forces at the contact interface are neglected. This simplification facilitates numerical stability and allows obtaining fast and accurate results in the preliminary phases of the analysis. For more detailed and realistic simulations, **Frictional** contacts can be employed, which account for the effects of friction.

The **Augmented Lagrange** method was used to solve the contact problem. This hybrid technique combines the advantages of the penalty method and Lagrange multipliers, ensuring the enforcement of the non-penetration condition between surfaces. Iterative adjustment of the Lagrange multipliers provides precision and robustness in computing the interaction between gear teeth.

Conclusion

The simulation of the actual meshing, through the construction of a precise geometric model and the analysis of the interaction between the gears, provides a detailed understanding of the stress distribution and structural behavior of the gear system. The use of the 18CrNiMo7-6 material, combined with detailed contact modeling, allows obtaining valid results that are useful for optimizing the gear design in power transmission applications.

Discretization and Contact Modeling

The simulation employed 20-node solid elements (SOLID186) for advanced discretization of the geometry, ensuring an accurate representation of the structural behavior. The contact elements used were CONTA174 (6-node) for the contacting surface and TARGE170 (8-node) for the target surface, enabling a detailed simulation of the interaction between components. These element types ensured a faithful capture of stress distribution and the contact phenomenon in the meshing zone.

Loading Conditions

For the application of loading conditions, the test pinion was fully constrained (all six degrees of freedom fixed), ensuring rigid fixation within the model. The mating gear was allowed to rotate only about its own axis, with a torque applied to it. This configuration was chosen to replicate as accurately as possible the real operating conditions in the meshing zone, allowing the analysis of mechanical behavior under load.

Experimental Results and Stress Comparison

To evaluate the stress distribution generated during meshing, the model was analyzed at six successive rotational positions, corresponding to the initial contact points on the reference tooth. These positions were selected to coincide with the sections previously defined according to the Dorn methodology, ensuring comparability of the results.

At each of these positions, the maximum stresses in the fillet region were measured at points labeled along the tooth profile (14–19), corresponding to critical loading zones. The results obtained from numerical simulation were compared with those determined analytically using the superposition of effects method, thus validating the numerical model.

Stress Distribution

The simulation provided detailed distributions of local stresses at various points on the tooth face (D1–D6), while the experimental stress values in the fillet region were recorded in MPa. These data provide essential information for evaluating mechanical behavior and assessing the risk of defects, such as fatigue or pitting.

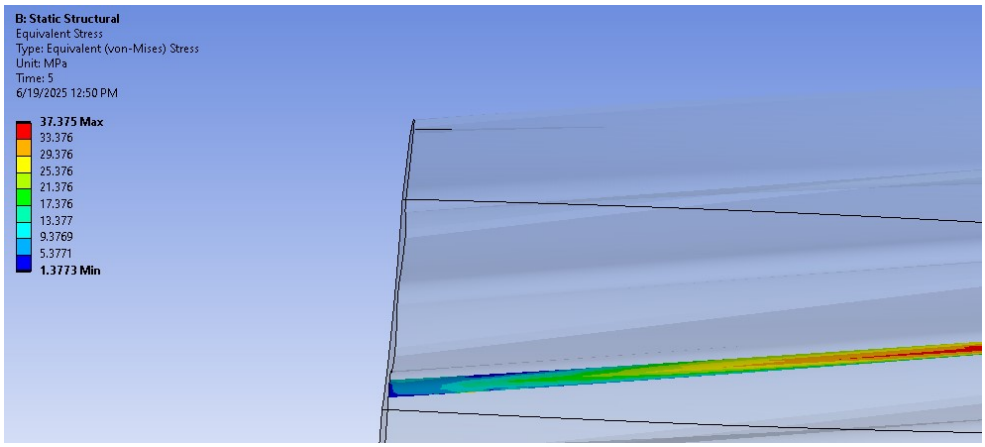


Fig. 8.5. Fillet Stresses of the Tested Tooth

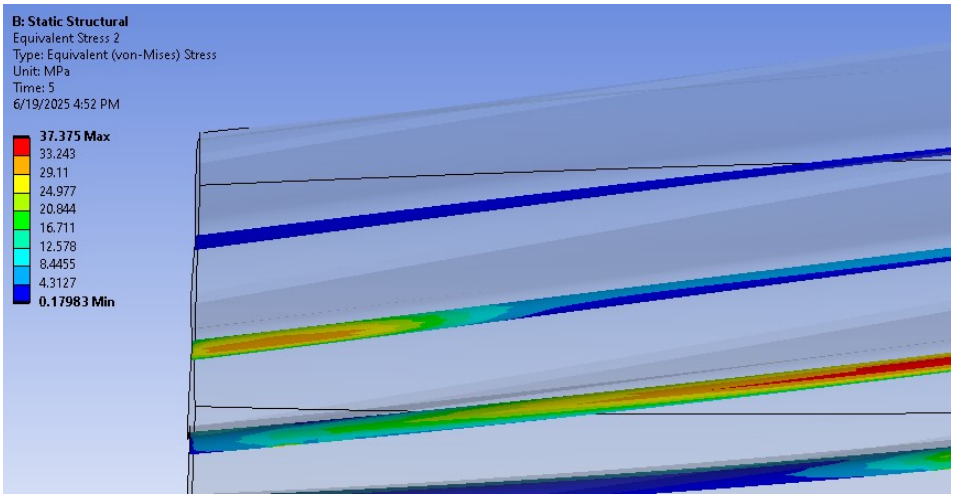


Fig. 8.6. Fillet Stress

Conclusion

The detailed numerical simulation of the tooth-to-tooth interaction and the stress distribution during meshing was validated by comparison with experimental results. This approach provided a comprehensive understanding of how the involute contact between teeth influences the stress state and the performance of the assembly, proving useful for optimizing the design and enhancing the durability of the gears.

Chapter 7. Influence Coefficient Method

Comparison Between Pin Loading and Real Tooth-by-Tooth Meshing

The significant differences in load transmission between pin loading and real tooth-by-tooth meshing are highlighted in the analysis of stresses at the tooth root. This region is less affected by local contact peculiarities and serves as a relevant reference point for evaluating structural stresses. To ensure a proper comparison between the two configurations, the stress distribution obtained from the pin-loading simulation was recalibrated using the influence coefficient method.

Fillet Stresses (Real Meshing at 760 N·m)

In the table above (Table 9.1), the fillet stresses are presented for various transmitted torque values (M), ranging from 15 Nm to 135 Nm, measured at six points along the tooth profile (P1–P6). These values are essential for analyzing the structural behavior of the tooth at different stages of loading.

Influence Coefficient Method and Helical Gear Meshes

In helical cylindrical gears, multiple-tooth engagement can occur, characterized by a variable overall contact ratio, which may range from 1 to 4. This ratio depends on

geometric parameters such as the normal module, helix angle, tooth width, and gear ratio. In this context, the load distribution along the contact lines is nonlinear and significantly affects the elastic behavior of the tooth.

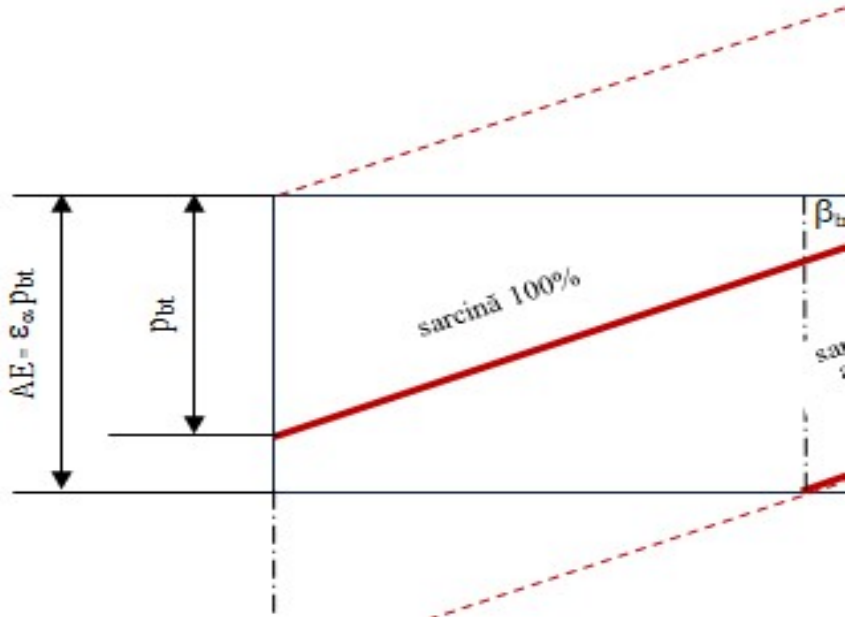


Fig. 9.3. Contact Configuration

It is important to note that the contact forces are not uniformly distributed along the entire contact line; rather, they follow an elliptical distribution, according to elasticity theory. This behavior is amplified by the "end effect," which manifests as a reduction of load towards the tooth flank edges. Consequently, deformations are concentrated at specific points along the tooth profile, and the analysis is performed based on a discretized model of concentrated loads.

Local Deformation and Elastic Behavior

The tooth deformation under load consists of two components: elastic bending deformation and local Hertzian-type deformation associated with elastic contact. When multiple pairs of teeth are engaged simultaneously, the load distribution and elastic interactions between flanks can no longer be accurately described using purely analytical methods. This necessitates the use of advanced numerical simulations, such as the Finite Element Method (FEM), to model the actual behavior of the gear.

Effective Loads and Multiple Contact Line Engagement

In the case of a helical cylindrical gear, the helix angle may allow simultaneous engagement on multiple contact lines. To accurately simulate real operating conditions, the load applied by the pin is considered equivalent to approximately 60% of the total

load corresponding to the transmitted torque, since the load distribution is not perfectly uniform due to manufacturing tolerances and assembly errors. This approach provides a more realistic representation of tooth behavior under multiple contact conditions.

Conclusions

Simulations using the influence coefficient method allowed a comparison between pin-loading and real gear meshing, highlighting the impact of load distribution and elastic interactions on tooth behavior. Additionally, the study emphasized the importance of advanced numerical simulations, such as FEM, to accurately capture the effects of deformations and stresses in helical gears with multiple lines of contact.

Chapter 8. Final Conclusions and Main Contributions on Modeling and Evaluating Fillet Stresses in Helical Cylindrical Gear Teeth Using FEM Simulation and Experimental Testing

This study analyzes the stress distribution in the fillet region of helical cylindrical gear teeth, comparing analytical calculations based on ISO and AGMA standards with results obtained through finite element analysis (FEM). A key finding is that standard methods significantly underestimate the maximum local stresses in the fillet region, highlighting the need for more detailed computational models.

Additionally, the geometry of the fillet and the presence of protuberances play a crucial role in stress propagation to significant depths, necessitating the inclusion of these geometric features in computational models. Consequently, a 2D geometric model, developed using a custom algorithm, was employed to realistically describe the tooth profile and correlated with CAD data.

In addition to the 2D analysis, 3D models proved valuable in highlighting new phenomena, such as the non-uniform distribution of stresses across the tooth width and the influence of the helix angle on the local stress state. These observations complement the 2D analysis and enable a more realistic description of the gear geometry behavior.

The analysis also emphasized the importance of heat treatments and the material's structural state on fatigue resistance. Specifically, surface hardness and the presence of compressive residual stresses help redistribute loads and delay crack initiation, indicating that an optimized design must be accompanied by appropriate heat treatments to achieve enhanced reliability.

The numerical simulations performed using FEM elements such as SOLID186, CONTA174, and TARGE170 showed that the tooth tip is not the critical area under the analyzed loading conditions; instead, the lever effect dominates the stress distribution along the flank. The simulation results were validated on an original experimental test rig, where loads were applied pointwise at controlled positions. Strain gauge measurements confirmed the trends identified in the numerical simulations, highlighting

the importance of calibration and interpolation for a comprehensive representation of the actual stress distribution.

Another key aspect of the study is the use of a higher-order polynomial interpolation method, which reduced experimental data processing errors and allowed extrapolation to different loading regimes. This method, with an error below 2%, can be integrated into computer-aided design software, providing a valuable tool for minimizing reliance on additional experimental tests.

The study also compares simplified static pin-loading models with FEM simulations of the actual dynamic gear engagement, highlighting the advantages and limitations of each method. Pin-loading models ensure experimental reproducibility but do not capture the complexity of multipoint contact interactions, whereas FEM simulations can accurately represent these effects but require significant computational resources. Correlating the two approaches provides an optimal balance between accuracy and efficiency.

Significant influences of geometric and technological parameters—such as the fillet radius, radial displacement coefficient, and surface quality—on the maximum stresses at the tooth root were also identified. These findings suggest that designs based solely on generalized standards are insufficient, and specific adjustments are necessary for industrial applications.

Regarding the scientific and technical contributions of the work, it includes the development of a proprietary code for generating the involute gear tooth profile with protuberance, the creation of a modular experimental rig for testing teeth under controlled loads, and the integration of a hybrid model validated by correlating FEM results, analytical calculations, and experimental measurements. Additionally, a fifth-order polynomial interpolation method was proposed for processing and generalizing experimental data.

Overall, the work contributes to the development of an advanced methodological framework for the mechanical analysis of helical cylindrical gears, with significant applications in applied research, industry, and education. It also opens new research directions, particularly in the development of computational and experimental models that include dynamic effects such as shocks, vibrations, and resonance, as well as modeling the influence of wear and micro-pitting on stress distribution. The use of advanced methods, such as damage mechanics or fatigue-life-based models, enables a more realistic and predictive assessment of the accumulated impact on the critical fillet zone.

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