NATIONAL UNIVERSITY OF SCIENCE AND
TECHNOLOGY POLITEHNICA BUCHAREST

DOCTORAL SCHOOL OF ELECTRICAL ENGINEERING

SUMMARY

DOCTORAL THESIS

CONTRIBUTIONS ON THE ADAPTATION OF IMPEDANCES IN ANALOGUE CIRCUITS AND COMMUNICATION NETWORKS

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List of abbreviations

SWR – Standing Wave Ratio

VSWR – Voltage Standing Wave Ratio

RTUS – Standing Wave Voltage Ratio

DUT – Device Under Test

VNA - Vector Network Analyzer

PR – Loss of Return

IL – Insertion Loss

RL – Return Loss

RF – Radio Frequency

CIRF – Radio Frequency Integrated Circuits

UTP – Unshielded Twisted Pair (cable used to transmit electrical signals)

ATU – Antenna Tuning Unit

SAC – Clockwise

SIAC – Counterclockwise

CMOS – Complementary Metal-Oxid-Semiconductor

SPICE – Simulation Program with Integrated Circuit Emphasis

ACAP – Analoque Circuit Analysis Program

SCAP – Symbolic Circuit Analysis Program

1. INTRODUCTION

In the context of the accelerated development of high-frequency communication systems, the design and optimization of RF (radio frequency) circuits and transmission lines is a major challenge for electronics engineers.

Impedance adaptation is a field of research of utmost importance in electrical engineering and modern communication technologies, and has been extensively studied in the literature, in the context of optimizing the performance of analog circuits and communication systems. The first approaches focused on traditional passive methods, using LC networks (coils and capacitors) to achieve adaptation, techniques that have remained relevant today due to their simplicity and stability. The fundamental concept underlying impedance matching is that of maximizing power transfer between source and load, achieved by matching the impedances of the two entities. This matching allows to reduce energy losses, minimize signal reflections and, implicitly, maintain the quality and integrity of the transmitted signal.

This paper is structured as follows:

Chapter 1 – Introduction, based on an extensive documentation focused on the subject of the doctoral thesis, the current state of the adaptation of the impedances and the use of the Smith diagram in the analysis, synthesis and design of analog circuits, the purpose and structure of this work were exposed.

Chapter 2 – Smith Circular Diagram, which describes the Smith diagram and presents the practical way of using the diagram for: analysis and design of analog circuits in the frequency domain, determination of the reflection coefficient, impedance and complex admittance, as well as determination of the standing wave ratio.

Chapter 3 – Impedance Adaptation in Radio Frequency Circuits, in which the impedance adaptation using transmission lines is presented, a correct definition of impedance adaptation in electrical engineering is given and the design and construction of impedance adaptation are presented, calculating with great accuracy the values of the parameters used in the construction of adaptation circuits.

Chapter 4 – *Design of impedance adaptation networks,* the most modern and efficient procedures for projecting and building impedance adaptation circuits and signal transmission networks are exposed.

Chapter 5 – Thevevenin, Norton and Hybrid Equivalent Circuits, which sets out the necessary and sufficient conditions to be met by uniport linear circuits in order to be substituted by Thévenin and Norton Circuits. It also presents the necessary and sufficient conditions for a uniport linear circuit to be replaced by an equivalent hybrid circuit. Several illustrative examples are highlighted to illustrate the veracity of the procedures developed.

Chapter 6 – Conclusions, which presents the general conclusions resulting from the research carried out on the adaptation of the impedances, the use of the Smith diagram as well as the equivalent Thévenin, Norton and Hybrid circuits.

Despite the current generalization of personal computers and powerful computers, these types of graphics tools are still useful nowadays. The authors believe that the use of the Smith graph by students is an important pedagogical tool, since many aspects of voltage, current, impedance, voltage standing wave ratio (VSWR), reflection coefficient, and design

problems for matching can be easily interpreted and well visualized using the Smith graph. Additionally, it is interesting to note that some types of modern laboratory equipment, such as network analyzers, still have a display that mimics the Smith chart. For all these reasons, the authors still use the Smith graph in their courses to explain several concepts related to transmission lines to students.

2. SMITH DIAGRAM

Electrical circuits with concentrated parameters are those formed by dipole circuit elements that are defined by modeling physical devices (components) by selecting one of the electrical or magnetic properties considered essential. For example, for a resistor, the voltage at the terminals depends at each moment on the intensity of the current at the same time and consequently it does not accumulate electrical or magnetic energy. Since an electric current produces a magnetic field, modeling a physical device through a resistor involves neglecting magnetic energy. In the case of a coil, only the magnetic field established by the current through the winding is taken into account, and the resistive effects in the winding conductor and the capacitive effects between the coils are neglected. Similarly, in the case of the live capacitor, only the electric field in the insulator between the fittings is considered and the resistive and inductive effect of the fittings is neglected.

The localization of electrical conduction phenomena in resistors, magnetic phenomena in coils and electrical phenomena in capacitors exclusively, allows the formation of circuits by connecting them to nodes with connection conductors supposedly perfectly conductive. In making this approximation, the conduction phenomena in coils and capacitors, the magnetic phenomena in resistors and capacitors and the capacitive phenomena in coils and resistors are neglected; secondly, the phenomena which accompany the current through the connecting conductors whose conductivity is in reality finite are neglected.

If the connection conductors are long enough to form lines of transmission of energy and electrical signals between parts of the circuit, these phenomena can no longer be neglected. Short power lines, i.e. lines whose length l is much shorter than the wavelength of the signal, can be modeled with circuit elements with concentrated parameters.

A power line whose length is comparable to the wavelength of the signal is called a long power line. The distribution of voltage and current on the line is described by partial differential equations and the line admits a scheme with uniformly distributed resistors, coils and capacitors (circuit with uniformly distributed parameters).

The theory of circuits with distributed parameters is also applied to the study of phenomena in different fields, such as: the study of overvoltages in transformers and in the windings of electrical cars, the modeling of electronic components (integrated circuits), the modeling of interconnection circuits in integrated circuits, as well as in the study of the phenomena of heat diffusion, electromagnetic field, mass transfer, etc.

In this chapter, the equations with partial derivatives of the first and second order - the equations of the telegraphers of long power lines - are established and the permanent harmonic regime is treated.

2.1. DESCRIPTION OF THE SMITH DIAGRAM

It was invented by P.H. Smith in 1939, while he was working for Bell Laboratory. The diagram is of overwhelming importance in the field of microwave circuit analysis, but also in the adaptation of impedances. In short, it is a complex polar representation that allows the visualization of impedance and admittance intuitively, the matching of the impedance

between a source and a load, the analysis of the reflection coefficient, but also the design of adaptation networks to maximize power transfer. Regardless of the values of impedances or reactants on the line, they can be represented on the diagram even if they tend towards infinity. The diagram compresses large impedance values through a coordinate transformation in which parameters such as: impedance, admittance and reflection coefficient are visualized graphically along the line. Thus, a new visualization perspective is obtained, through a restricted graphic representation, having a permanent overview of the behavior of the transmission system (in terms of its performance through the line parameters) in close correlation with the working frequency. To better understand this approach, it is like comparing a watch with a digital display that measures the time exactly with a classic one, with a dial, which displays both the current time and the time remaining or elapsed since a considered event, which visually induces a more complete overview of the passage of time. and increases the degree of temporal perception. The Smith diagram materialized from the need to graph the very large and very small values in a finite space, and to increase the speed of calculation at a time when there were no high-performance portable computers, [1-22].

The Smith diagram is one of the most widely used graphical procedures for high-frequency circuit applications. The Smith diagram can be used to display multiple parameters simultaneously, including impedances, admittances, reflection coefficients, scattering parameters, noise circles, constant gain contours, and regions for unconditional stability, including mechanical vibration analysis.

The Smith Diagram provides a smart way to visualize complex functions and continues to endure its popularity for decades after the concept was discovered. While the use of paper Smith diagrams to solve the complex mathematics involved in matching problems has largely been replaced by software-based methods, the Smith diagram is still a very useful method of showing how RF parameters behave at one or more frequencies, an alternative to using tabular information. Thus, most RF circuit analysis software includes a Smith diagram option for displaying results, and all but the simplest impedance measurement instruments can represent the measured results on a Smith diagram display, [1-22].

The Smith diagram is a mathematical transformation of the two-dimensional Cartesian complex plane. Map of complex numbers with positive real parts inside the circle. Those with negative real parts are represented outside the circle. If we are dealing only with resistance and positive reactances, our interest is focused on the area inside the circle.

The Smith diagram is plotted on the complex plane of the reflection coefficient in two dimensions and is scaled to normalized impedance (the most common), normalized admittance, or both, using different colors to distinguish them. These are often known as Z, Y, and YZ Smith charts. Normalized scaling allows the Smith diagram to be used for problems involving any system feature or impedance that is represented by the center point of the diagram. The most commonly used normalizing impedance is 50 ohms. Once an answer is obtained through the graphical constructs described below, it is simple to convert between the normalized impedance (or normalized admittance) and the corresponding unnormalized value by multiplying by the characteristic impedance (admittance). Reflection coefficients can be read directly from the diagram because they are dimensionless parameters with no units (dimensionless).

The Smith diagram has a scale around its circumference or periphery, which is graded in wavelengths and degrees. The wavelength scale is used in distributed component problems and represents the distance measured along the connected transmission line between the generator or source and the load to the point considered. The degree scale represents the angle of the coefficient of reflection of the tension at that point. The Smith chart can also be used for problems of matching and analyzing aggregate items.

Thus, the Smith diagram is a 2-D representation of all possible complex impedances with respect to the coordinates defined by the complex reflection coefficient. The definition range of the reflection coefficient for a lossless line is a circle with a unit radius in the complex plane (Fig. 2.1, (a)). In the general case of a lossy transmission line, the reflection coefficient could have an amplitude greater than one, due to the characteristic impedance structure, which requires an extension of the Smith diagram [1-23].

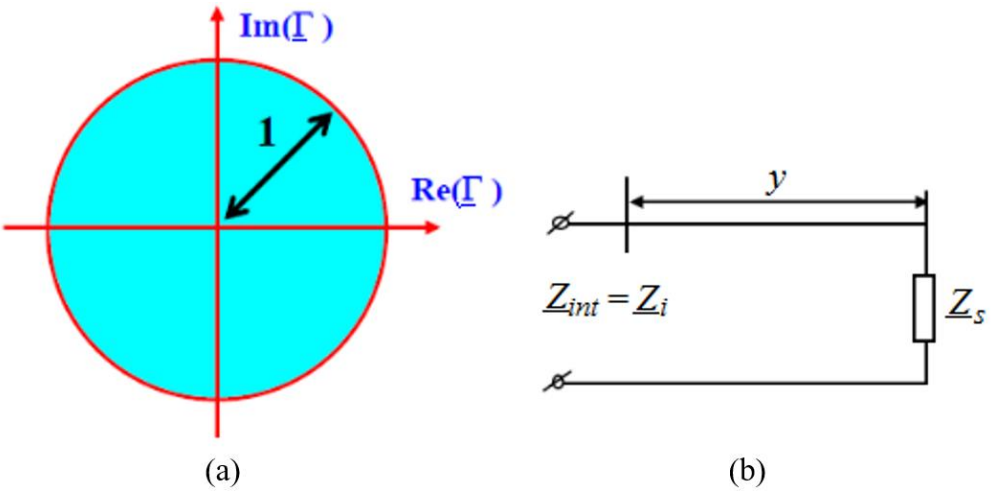


Figure 2.1. (a) The scope of definition of the reflection coefficient for a lossless line; (b) Definition of complex input impedance.

The solutions of the telegraph equations present the voltages and currents analyzed at the distance y from the load, so that we can replace the assembly formed by the load impedance $\underline{Zs}$ and the line section with an impedance, called the input impedance (fig. 2.1, (b)), [1-12].

$$\underline{Z}_{int} = \underline{Z}_i = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}_s ch(\underline{\gamma}_l y) + \underline{Z}_c \underline{I}_s sh(\underline{\gamma}_l y)}{\frac{\underline{U}_s}{\underline{Z}_c} sh(\underline{\gamma}_l y) + \underline{I}_s ch(\underline{\gamma}_l y)} \tag{2.1}$$

Dividing the numerator and denominator of the relationship (2.1) with , we get:$\underline{I}_s ch\left(\underline{\gamma}_l y\right)$

$$\underline{Z}_i = \frac{\underline{U}}{\underline{I}} = \underline{Z}_c \frac{\underline{Z}_s + \underline{Z}_c th(\underline{\gamma}_l y)}{\underline{Z}_c + \underline{Z}_s th(\underline{\gamma}_l y)} \tag{2.2}$$

Expressing the voltage and current as a function of the direct and reflected (inverse) quantities, the result is:

$$\underline{Z}_i = \frac{\underline{U}}{\underline{I}} = \frac{\underline{U}_d + \underline{U}_i}{\underline{I}_d - \underline{I}_i} = \frac{\underline{U}_d(1+\underline{\Gamma})}{\underline{I}_d(1-\underline{\Gamma})} = \underline{Z}_c \frac{(1+\underline{\Gamma})}{(1-\underline{\Gamma})}. \tag{2.3}$$

For the simplicity of the calculations, the normalized complex impedances are used, i.e. they are divided by *Zc* (*day* = *Day*/*Zc* and *zs* = *Zs*/*Zc*). Obviously, the characteristic impedance becomes *Zc*/ *Zc* = 1.

In the following we will use normalized impedances (related to Z_c=Z0) and denoted in lowercase letters. As soon as we notice that the characteristic impedance of a line is unitary, we will understand that the other quantities of the scheme are also normalized (and therefore dimensionless). Denormalization, the return from normalized to non-normalized quantities, is achieved by multiplying by the normalization impedance.

Therefore, we can express the reflective coefficient as a function of the normalized impedance $\underline{\Gamma}(y)\underline{z} = \underline{z}(y)$ and vice versa, according to the relations:

$$\underline{\Gamma}(y) = \frac{\underline{z}(y)-1}{\underline{z}(y)+1} \qquad \text{and} \qquad \underline{z}(y) = \frac{1+\underline{\Gamma}(y)}{1-\underline{\Gamma}(y)} \tag{2.4}$$

Equations (2.4) describe a one-to-one correspondence between the z-coefficients and $\underline{\Gamma}$. This means that each normalized impedance corresponds to a single point in the complex plane of $\underline{\Gamma}$. For example, we want to mark or somehow identify the values of normalized impedance $\underline{z}$ that correspond to different points in the complex plane $\underline{\Gamma}$. Table 2.1 presents some cases of one-to-one correspondence between complex impedances and the complex plane $\underline{\Gamma}$, [1-19].

Table 2.1 Particular cases of one-to-one correspondence between complex impedances and the Γ plane

If	$\underline{Z}$	$\underline{z}$	$\underline{\Gamma}$
1	∞	∞	1
2	*0*	*0*	-1
3	$\underline{Z}_c$	*1*	*0*
4	$j\underline{Z}_c$	*j*	*j*
5	$-j\underline{Z}_c$	*-j*	*-j*

Therefore, it can be seen that the five normalized complex impedances correspond biunivocally to five different points in the complex plane $\underline{\Gamma}$ (fig. 2.2) and, respectively, the five different points in the complex plane $\underline{\Gamma}$ correspond biunivocally to five different values of the normalized complex impedances (fig. 2.3), [11-23].

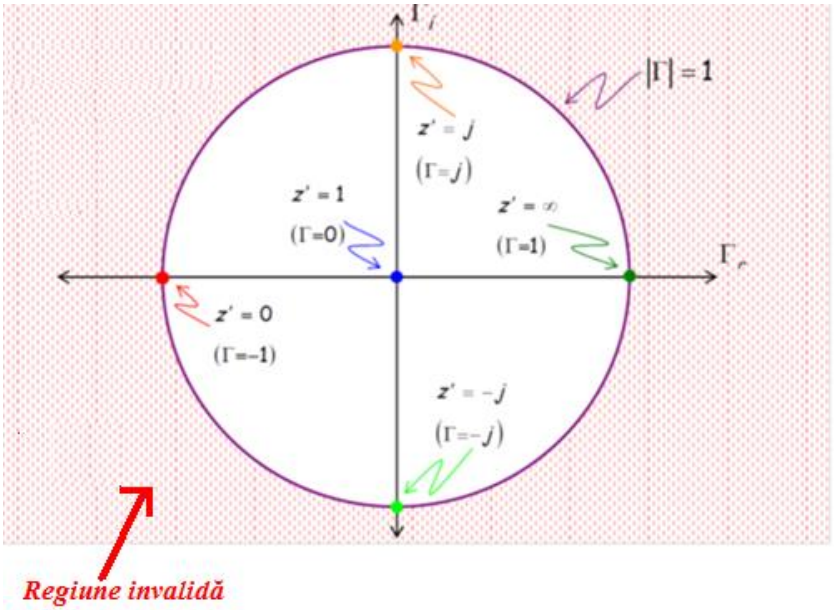


Figure 2.2. The correspondence between the values of the normalized complex impedances and the points in the complex plane $\underline{\Gamma}$

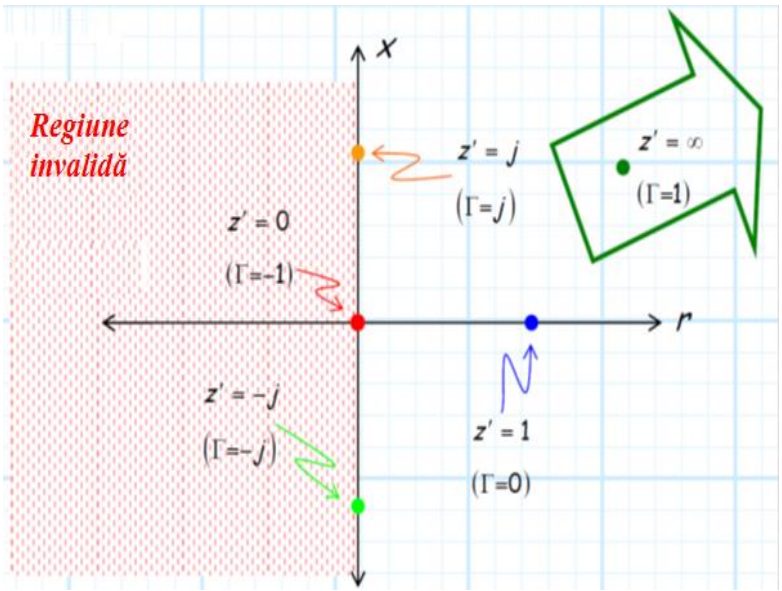


Figure 2.3. Correspondence between points in the complex plane $\underline{\Gamma}$ and values of normalized complex impedances

The Smith diagram provides a graphical representation of Γ that allows the determination of quantities such as VSWR or the output (termination) impedance of a device under test - DST (Device Under Test - DUT). It uses a bilinear Moebius transformation, projecting the complex impedance plane onto the complex plane $\underline{\Gamma}$:

$$\underline{\Gamma}(y) = \frac{\underline{z}(y)-1}{\underline{z}(y)+1} \quad \text{and} \quad \underline{z}(y) = \frac{1+\underline{\Gamma}(y)}{1-\underline{\Gamma}(y)} \tag{2.5}$$

As can be seen in figure 2.4, the half of the plane with the positive real part of the impedance $\underline{Z}$ is transposed inside the unitary circle of the plane $\underline{\Gamma}$.

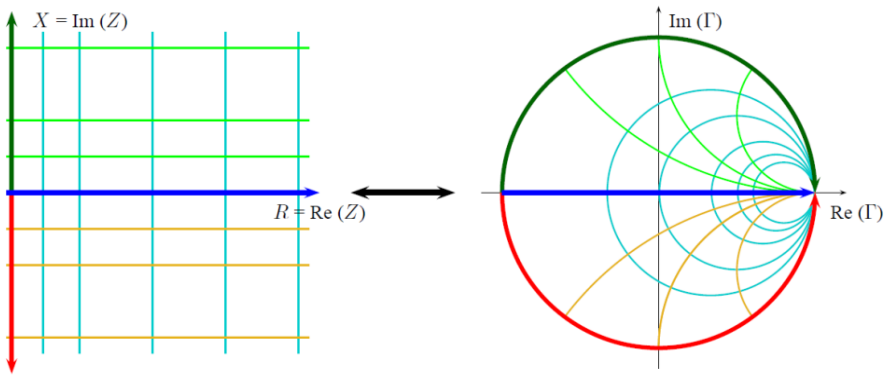


Figure 2.4. Illustration of the Moebius transform from the complex impedance plane to the plane $\underline{\Gamma}$ commonly known as the Smith diagram

The paper presents some examples of biunivocal correspondence between points in the plane of normalized complex impedance and the complex plane $\underline{\Gamma}$.

When the impedance considered is purely real $\underline{Z} = R$ ($\mathrm{Im}\{\underline{Z}\} = 0$). For the normalized complex impedance $\underline{z}$, we have $\underline{z} = r + \mathrm{j}.0$ (i.e. $\mathrm{Im}\{z\} = 0$ and $r = R/|\underline{Z}_c|$). In this case, the reflection coefficient $\underline{\Gamma}$ has the expression:

$$\underline{\Gamma} = \frac{r-1}{r+1} \tag{2.6}$$

that is: $\Gamma_r = Re\{\underline{\Gamma}\} = \frac{r-1}{r+2}$ and $.\Gamma_i = Im\{\underline{\Gamma}\} = 0$

Thus, we determine a two-way correspondence between two contours – one contour (x = 0) on the normalized complex impedance plane and the other on the complex plane $\underline{\Gamma}$ (Figure 2.5).

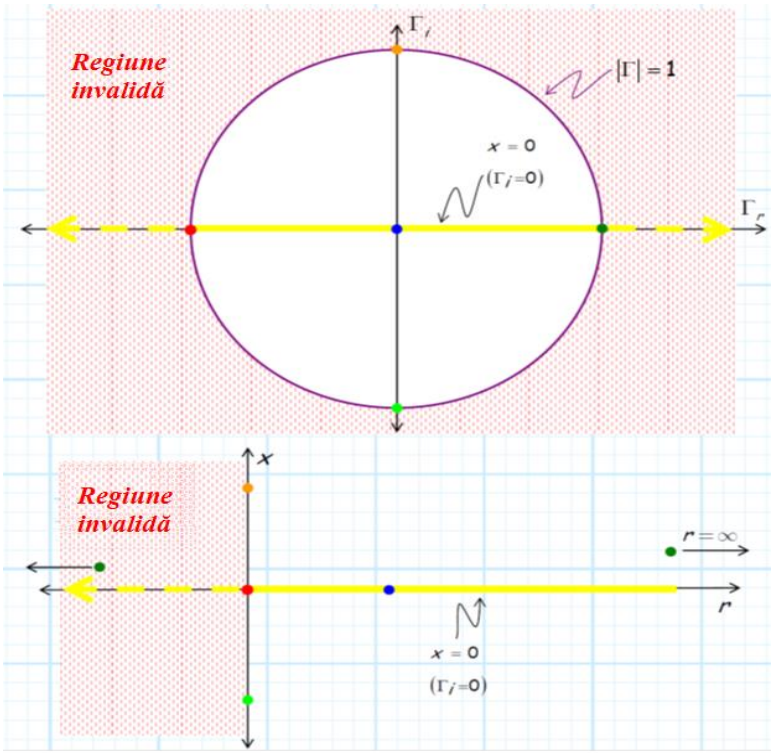


Figure 2.5. Biunivocal correspondence between two contours – one contour ($x = 0$) on the plane of normalized complex impedance and the other on the complex plane $\underline{\Gamma}$

When the impedance considered is purely imaginary $\underline{Z} = jX$ ($\mathrm{Re}\{\underline{Z}\} = R = 0$), i.e. the impedance has no resistive component. In this case, the normalized complex impedance has the expression:

$$\underline{z} = 0 + jx \ (r = 0) \tag{2.7}$$

where $x = X/|\underline{Z}_c|$

The purely imaginary normalized complex impedance corresponds to a reflection coefficient with the magnitude unit, $|\underline{\Gamma}| = 1$, in the complex plane $\underline{\Gamma}$ (Figure 2.6).

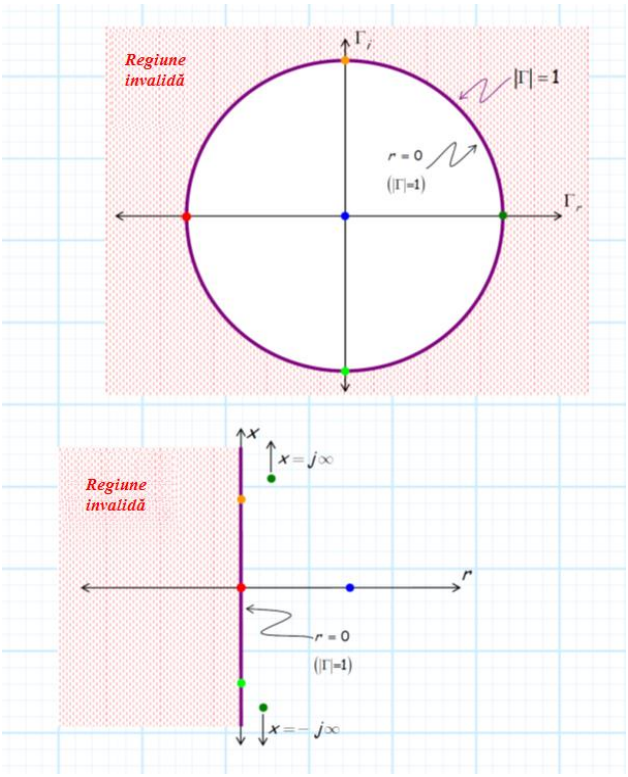


Figure 2.6. Biunivocal correspondence between two contours – one contour (r = 0) on the plane of the normalized complex impedance and the other ($|\Gamma|$, a circle with center in origin and unit radius) on the complex plane Γ

Thus, a two-way correspondence is determined between two contours – one contour ($r = 0$) on the plane of the normalized complex impedance and the other $|\underline{\Gamma}| = 1$, a circle with center in origin and radius unit) on the complex plane $\underline{\Gamma}$.

2.2. CONCLUSIONS

The usefulness of the Smith diagram is outstanding in a wide range of applications, from matching the impedances of a small signal amplifier for minimal noise, to performing a Load Pull analysis for a power amplifier. In the latter case, a power amplifier is forced to work on various impedances, at which the phase is rotated 360 degrees. The output power is then figured on the Smith diagram and allows the optimal output impedance for that

transistor/amplifier to be determined, in order to obtain the maximum power, maximum linearity or maximum efficiency.

The usefulness of the Smith diagram is special not only for professionals in the field, but also for radio amateurs who increasingly use modern tools for the analysis of circuits or antennas. Whether it is antenna analyzers for radio amateurs such as SARK 110, RigExpert AA55, KC901 or a mini-VNA (Vector Network Analyzer), the correct understanding of the use of the Smith diagram allows the user to see exactly what the nature of the impedance is and implicitly what is the real (resistive) and the imaginary (reactive) component. Starting from this, conclusions can be drawn whether the antenna is too short or too long and conclusions related to how the adaptation circuit should be built.

The diagram provides a compact and handy representation of all passive impedances (passive impedances are positive real part impedances) from 0 to ∞. Impedances with negative real parts, such as the reflection amplifier or any other active device, would appear outside the Smith diagram.

Since mapping converts impedances or admittances ($\underline{y} = 1/\underline{z}$) into reflection factors and vice versa, it is particularly interesting for radio frequency and microwave studies.

The transition between impedance and intake in the diagram is particularly easy: $\underline{\Gamma}(\underline{y} = 1/\underline{z}) = -\underline{\Gamma}(z)$;

Moreover, the reference plane in the Smith diagram can be moved very easily by adding a transmission line of the correct length. Many Smith charts have rulers below the complex plane $\underline{\Gamma}$ from which a variety of quantities can be determined.

3. ADAPTING IMPEDANCES IN RADIO FREQUENCY CIRCUITS

3.1. INTRODUCTION

In electrical engineering, impedance matching is the practice of designing the input impedance of an electrical load or the output impedance of the corresponding signal source to maximize power transfer or minimize signal reflection from the load. An electrical power source, such as a generator, amplifier, or radio transmitter, has a source impedance equivalent to an electrical resistance in series with a frequency-dependent reactance. Likewise, an electrical charge, such as a light bulb, transmission line, or antenna, has an impedance equivalent to a resistance in series with a reactance.

The maximum active power transfer theorem states that the maximum active power is transferred from the source to the load when the load resistance is equal to the resistance of the source and the load reactance is equal (with the negative of the source reactance) to the reactance of the switched sign source: the reactances cancel each other out with their opposite frequency dependence. Another way to say this using complex numbers is that the load impedance must be equal to the complex conjugate of the source impedance. If this condition is met, the two parts of the circuit are said to be impedance-matched (impedance-adapted).

In a direct current (dc) circuit, the condition is met if the load resistance is equal to the resistance of the source. In an alternating current (AC) circuit, the reactance depends on the

frequency, so circuits that are impedance-matched to a frequency may not be impedance-matched if the frequency is changed. Matching impedance over a wide frequency band will generally require complex, filter-like structures with many components, except in the mundane case of constant source and load resistances, when a transformer can be used.

In the case of a complex source of $\underline{ZS}$ impedance and $\underline{ZL}$ load, the maximum power transfer is achieved when:

$$\underline{Z}_S = \underline{Z}_L^*, \tag{3.1}$$

where asterisk indicates the complex conjugate of the variable. Where $\underline{ZS}$ is the characteristic impedance of a transmission line, the minimum reflection is obtained when:

$$\underline{Z}_S = \underline{Z}_L \tag{3.2}$$

When the adaptation condition (3.2) is met, it means that we have no reflected wave from the load. A measure of adaptation can also be given by the *parameter PR - Return Loss (RL) which* measures the attenuation of the signal reflected by the load, [1]:

$$PR(dB) = 20\, log_{10} \frac{P_r}{P_i} \tag{3.3}$$

where *Pi* is the power of the wave incident on the load and *Pr* is the power of the wave reflected by the load. In the ideal case of a perfectly matched load, the PR parameter tends to -∞. Taking into account that the powers of the forward and reverse waves are proportional to the square of the amplitudes, then we can write the PR parameter and according to the reflection coefficient Γ, [1]:

$$PR(dB) = 20\, log_{10} \frac{|\underline{U}^-|}{|\underline{U}^+|} = 20\, log_{10} |\underline{\Gamma}| \tag{3.4}$$

If we introduce a new parameter it will give us an idea of how selective the adaptation is, in other words, how large the spectral range over which the adaptation is carried out. We set the limit of the mismatch at -30dB, i.e. we will consider that the adaptation is achieved if:

$$|PR(dB)| > 30 \text{ dB} \tag{3.5}$$

The insertion loss is the ratio of the transmitted signal power level to the input signal power level expressed in decibels (dB), i.e. [1-20]:

$$IL(dB) = 10\ log\, 1\, 0(P_i/P_t),$$
$$P_i = P_t + P_r;\ P_t/P_i + P_r/P_i = 1 \tag{3.6}$$

Figure 3.1. shows an example of return loss and insertion loss in a lossless circuit or on a transmission line, [1-20].

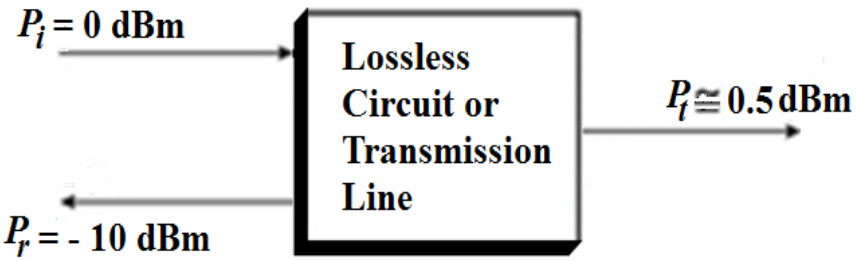


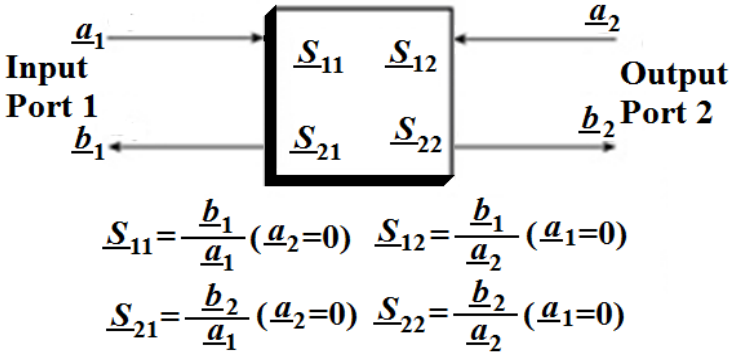
Figure 3.1. Return loss and insertion loss in a lossless circuit or transmission line

In Figure 3.1, a 0-dBm signal, *Pi*, is applied to the transmission line. The reflected power, *Pr*, is displayed as -10 dBm, and the return loss is 10 dB. The higher the value, the better the fit, i.e. for a perfect fit, the return loss, ideally, is ∞, but a return loss of 35 to 45 dB is usually considered a good fit. Similarly, for an open circuit or a short circuit, the incident power is reflected back. The return loss for these cases is 0 dB, [1-20].

Referring to Figure 3.1, *Pr* of -10 dBm means that 10% of the incident power is reflected. If the circuit or transmission line is lossless, 90% of the incident power is transmitted. The insertion loss is therefore about 0.5 dB, resulting in a transmitted power of -0.5 dBm. If there were internal losses, the insertion loss would be greater.

Using the S *parameters*, the performance of a radio frequency (RF) circuit can be fully characterized without the need to know its internal composition. For these purposes, the circuit is commonly referred to as a "black box". The internal components can be active (i.e. amplifiers) or passive. The only provisions are that the *S-parameters* are determined for all frequencies and conditions (e.g., temperature, amplifier polarization) of interest, and that the circuit must be linear (i.e., its output is directly proportional to its input), [1-20].

Figure 3.2 is a representation of a simple microwave circuit with an input and output (called ports). Each port has an incident signal (a) and a reflected signal (b). By knowing the *S* parameters (i.e. *S11*, *S21*, *S12*, *S22*) of this circuit, one can determine its effect on any system in which it is installed, [1-20].



$$S_{11}=\frac{b_1}{a_1}(a_2=0) \quad S_{12}=\frac{b_1}{a_2}(a_1=0)$$

$$S_{21}=\frac{b_2}{a_1}(a_2=0) \quad S_{22}=\frac{b_2}{a_2}(a_1=0)$$

Figure 3.2. S-parameter representation of a two-port microwave circuit.

The *S-parameters* are determined by measurement under controlled conditions. Using special test equipment called a network analyzer, a signal (*a1*) is fed into port 1 with port 2 terminated in an impedance-controlled system (usually 50 ohms). The analyzer simultaneously measures and records *a1*, *b1,* and *b2* (*a2* = 0). The process is then reversed, i.e. with a signal (*a2*) entered in port 2, the analyzer measures *a2, b2 and b1* (*a1* = 0). In its simplest form, the network analyzer only measures the amplitudes of these signals. This is called a scalar network analyzer and is sufficient for determining quantities such as the Voltage Standing Wave Ratio (VSWR), *RL* and *IL.* For complete circuit characterization, however, phase is also required and requires the use of a vector network analyzer. The S-parameters are determined by the following relations, [1-20]:

$$S_{11}=\left.\frac{b_1}{a_1}\right|_{a_2=0};\ S_{21}=\left.\frac{b_2}{a_1}\right|_{a_2=0};\ S_{22}=\left.\frac{b_2}{a_2}\right|_{a_1=0};\ S_{12}=\left.\frac{b_1}{a_2}\right|_{a_1=0} \tag{3.7}$$

The parameters *S11* and *S22* are the reflection coefficients of the input and output port, respectively; while *S21* and *S12* are the forward and backward transmission coefficients of the circuit [1-22]. The return loss *RL* is related to the reflection coefficients through the relationships:

$$RL_{Portul1}(dB) = -20\,log\,1\,0\left(\left|\underline{S}_{11}\right|\right) \text{ si } RL_{Portul2}(dB) = -20\,log\,1\,0\left(\left|\underline{S}_{22}\right|\right)\left| \quad (3.8)\right.$$

The insertion *loss* *IL* is related to the transmission coefficients of the circuits by the relations:

$$IL_{Portul1 \text{ la Portul2}}(dB) = -20\,log\,1\,0\left(\left|\underline{S}_{21}\right|\right) \text{ si } L_{Portul2 \text{ la Portul1}}(dB) = -20\,log\,1\,0\left(\left|\underline{S}_{12}\right|\right)\left| \quad (3.9)\right.$$

This representation can be extended to microwave circuits with an arbitrary number of ports. The number of *S-parameters* increases with the square of the number of ports, so the mathematics becomes more involved, but it is easy to use with the help of matrix algebra, [1-20].

Figure 3.3 shows the source and complex impedance of the load circuit:

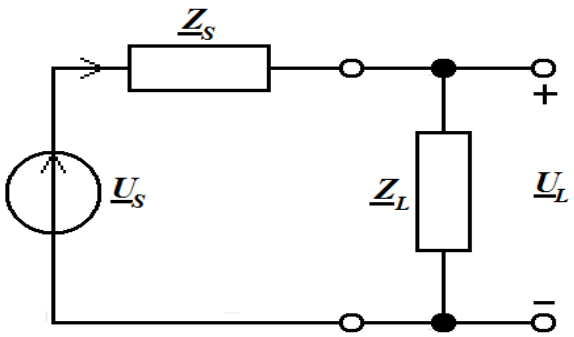


Figure 3.3. Source and impedance of the load circuit

We consider the frequency band (B) around the center frequency f_0 for which condition (3.5) is realized, i.e. the difference between the maximum frequency (f_2) and the minimum frequency (f_1) for which the modulus of the parameter PR is greater than 30 dB. We can then define the quality factor of the adaptation circuit as, [1]:

$$Q = \frac{f_0}{B} = \frac{f_2+f_1}{2(f_2-f_1)} \quad (3.10)$$

The concept of impedance matching found its first applications in electrical engineering, but it is also relevant in other applications where a form of energy, not necessarily electrical, is transferred between a source and a load. An alternative to impedance matching is the impedance bridge, where the load impedance is chosen to be much higher than the source impedance and the primary goal is to maximize voltage transfer rather than power.

Impedance is the opposition of a system to the flow of energy from a source. For constant signals, this impedance can also be constant. For variable signals, they usually change with frequency. The energy involved can be electrical, mechanical, acoustic, magnetic, optical, or thermal. The concept of electrical impedance is perhaps the best known.

The modulus of the electrical impedance, like the electrical resistance, is measured in ohms. In general, impedance has a complex value; this means that charges generally have a resistance component (symbol: R) that forms the real part of $\underline{Z}$ and a reactance component (symbol: X) that forms the imaginary part of $\underline{Z}$. In simple cases (such as low-frequency or direct current power transmission) the reactance may be negligible or non-existent; Impedance can be considered a pure resistance, expressed by a real number. In the following we will consider the general case in which resistance and reactance are both significant and the special case in which reactance is negligible.

Tuning the impedance to minimize reflections is achieved by making the load impedance equal to the source impedance. If the source impedance, load impedance, and characteristic impedance of the transmission line are purely resistive, then the non-reflective adaptation is the same as the match of the maximum power transfer.

Whenever a power source with a fixed output impedance, such as an electrical signal source, radio transmitter, or mechanical sound (e.g., a loudspeaker) operates in a load, the maximum possible power is supplied to the load when the load impedance (load impedance or input impedance) is equal to the complex conjugate of the source impedance (i.e., its internal impedance or output impedance). For two impedances to be conjugated complexes, their resistances must be equal, and their reactances must be equal in magnitude, but of opposite signs. In low-frequency or dc systems (or systems with purely resistive sources and charges) the reactances are zero or small enough to be ignored. In this case, the maximum power transfer occurs when the resistance of the load is equal to the resistance of the source (see the maximum power theorem for a mathematical proof).

Impedance matching is not always necessary. For example, if a source with a low impedance is connected to a load with a high impedance, the power that can pass through the connection is limited by the higher impedance. This maximum voltage connection is a common configuration called an impedance bridge or voltage bridge, and it is widely used in signal processing. In such applications, providing a high voltage (to minimize signal degradation during transmission or to consume less energy by reducing currents) is often more important than maximum power transfer.

In older audio systems (based on transformers and passive filter networks and based on the telephone system), the source and load resistances were equalized to 600 ohms. One reason for this was to maximize power transfer, as there were no amplifiers available that could restore the lost signal. Another reason was to ensure the proper operation of the hybrid transformers used in central exchange equipment to separate the output from the input speech so that they could be amplified or fed to a four-wire circuit. Most modern audio circuits, on the other hand, use active amplification and filtering and can use voltage bridge-type connections for the highest accuracy. Strictly speaking, impedance matching only applies when the source and load devices are linear; however, matching can be achieved between nonlinear devices within certain operating ranges.

3.2. IMPEDANCE MATCHING DEVICES

Adjusting source impedance or load impedance, in general, is called "impedance matching." There are three ways to improve an impedance mismatch, all of which are called "impedance matching":

- Devices intended to exhibit an apparent source load $\underline{Zload} = \underline{Z}^*_{source}$ (conjugate complex adaptation). Given a source with a fixed voltage $\underline{US}$ and a fixed impedance $\underline{ZS}$ of the source, the maximum power transfer theorem says that this is the only way to extract the maximum power from the source;
- Devices intended to exhibit an apparent charge of $\underline{Zload} = \underline{Zline}$ (complex impedance adaptation) to avoid echoes. Given a transmission line source with a fixed impedance, this "non-reflective impedance matching" at the end of the transmission line is the only way to avoid reflecting echoes back to the transmission line;
- Devices intended to exhibit an apparent source resistance as close to zero as possible, or having as high an apparent source voltage as possible. This is the only way to maximize energy efficiency and is therefore used at the beginning of power lines. Such an impedance bridge connection also minimizes electromagnetic distortion and interference; It is also used in modern audio amplifiers and signal processing devices.

There are a variety of devices used between a power source and a load that perform "impedance matching." To adapt to electrical impedances, engineers use combinations of transformers, resistors, coils, capacitors, and transmission lines. These passive (and active) impedance matching devices are optimized for different applications and include baluns (devices for UTP cable video transmission), antenna tuners (sometimes called ATUs or roller-coasters), acoustic horns, matching networks, and terminators.

There are several ways to do impedance matching, including:

- Computer simulations: Complex but simple to use, since such simulators are dedicated to different design functions and not to impedance matching. Designers must be familiar with the multiple data entries that need to be entered and the correct formats. They also need expertise to find the useful data among the tons of results that come out. In addition, circuit simulation software is not pre-installed on computers, unless they are dedicated to such an application;
- Manual calculations: tedious due to the length ("kilometers") of the equations and the complex nature of the numbers to be manipulated;
- Instinct: This can only be acquired after dedicating many years to the RF industry. In short, this is for the super-specialist;
- Smith diagram.

3.2.1. Transformers

Transformers are sometimes used to match the impedances of circuits. A transformer converts alternating current at one voltage in the same waveform to another voltage. The input power to the transformer and output to the transformer is the same (except for conversion losses). The part with the lower voltage is with (at) low impedance (because it has a lower number of turns), and the part with the higher voltage is with (at) a higher impedance (because it has more turns in the coil).

An example of this method involves a television balun (rocking) transformer. This transformer converts a balanced signal from the antenna (via dual 300 ohm cable) into an unbalanced signal (75 ohm coaxial cable, such as the RG-6). To match the impedances of both devices, both cables must be connected to a suitable transformer with a turn ratio of 2 (such as a 2:1 transformer). In this example, the 75 ohm cable is connected to the transformer side with fewer turns; The 300 ohm line is connected to the multi-turn transformer part. The formula for calculating the transformer turn ratio for this example is:

$$\text{raportul spirelor} = \sqrt{\frac{\text{rezistența sursei}}{\text{rezistența sarcinii}}} \tag{3.11}$$

Resistive impedance adaptations are the easiest to design and can be made with a simple L (L pad) consisting of two resistors. Power loss is an inevitable consequence of the use of resistive networks and they are (usually) only used to transfer signals at the line level.

3.2.2. Step transmission lines

Most devices with concentrated elements can adapt (match) to a specific range of load impedances. For example, to adapt an inductive load to a real impedance, a capacitor must be used. If the load impedance becomes capacitive, the adapter element must be replaced with a coil. In many cases, it is necessary to use the same circuit to accommodate a wide range of load impedances, thus simplifying circuit design. This problem has been addressed by stepped transmission line,[3] in which quarter-wave dielectric interlinings, placed in series, are used to vary the characteristic impedance of a transmission line. By controlling the position of each element, a wide range of load impedances can be adapted without the need to reconnect the circuit.

3.3.3. Filter

Filters are commonly used to achieve impedance matching in telecommunications and radio engineering. In general, it is theoretically not possible to achieve a perfect impedance adaptation to all frequencies with a discrete component network. Impedance matching

networks are designed with a defined bandwidth, take the form of a filter, and use filter theory in their design.

Applications that require only narrow bandwidth, such as tuning filters (tuners) and radio transmitters, can use a simple tuned filter, such as a shunt (stub). This would provide a perfect fit only at a certain frequency. Broadband adaptation requires multi-section filters.

A stub method or simply stub [1,2] in software development is a piece of code used to replace other programming functionality. A stub can simulate the behavior of existing code (such as a procedure on a remote machine; such methods are often called simulations) or it can be a temporary substitute for code that has not yet been developed. Therefore, stubs are most useful in porting, distributed computing, as well as general software development and testing.

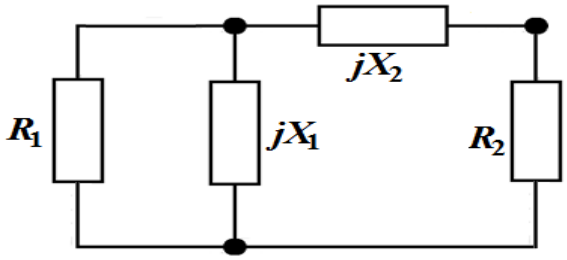


Figure 3.4. Basic scheme for adapting R1 to R2 with an L buffer. R1 > R2, however, R1 can be considered the source and$_{R2}$ can be the load. One of X_1 or X_2 must be a coil (an inductor), and the other must be a capacitor.

Figure 3.4 shows the basic scheme for adapting R_1 to R2 with an L buffer.
R1 > R2, however, R1 can be considered the source, and R2 can be the load. One of X_1 or X_2 must be a coil (an inductor), and the other must be a capacitor.

$$\frac{jR_1X_1}{R_1+jX_1} \Rightarrow R_2 = \frac{R_1X_1^2}{R_1^2+X_1^2} \tag{3.12}$$

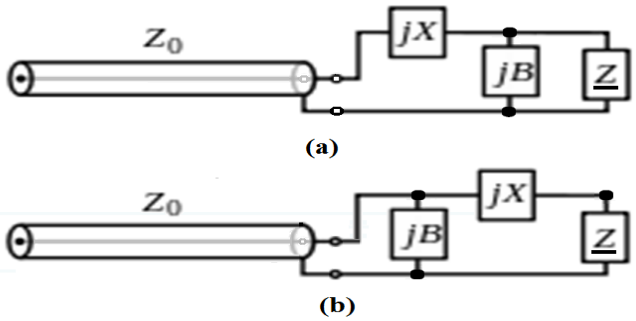


Figure 3.5. L networks for matching a source or load impedance Z to a transmission line with characteristic impedance Z_0. X and B can each be positive (coil) or negative (capacitor). If Z/Z_0 is inside the circle 1+jx on the Smith diagram (i.e. if $Re(Z/Z_0)>1$), the network (a) can be used; otherwise, the network (b), [4] can be used.

A simple electrical impedance matching network requires a capacitor and an inductor. In Figure 5,$_{R1}$ > R2, however, either$_{R1}$ or R2 may be the source and the other the load. One of X1 or$_{X2}$ must be an inductor, and the other must be a capacitor. One reactance is in parallel with the source (or charge), and the other is in series with the charge (or source) [1]. If a reactance is parallel to the source, the actual network adapts from high to low impedance.

The analysis is as follows [5]: we consider a real impedance of the source R_1 and a real load impedance of R_2. If an X1 reactance is parallel to the source impedance, the combined impedance can be written as:

$$\frac{jR_1X_1}{R_1+jX_1} \tag{3.13}$$

If the imaginary part of the above impedance is canceled by the series reactance, the real part is:

$$R_2 = \frac{R_1X_1^2}{R_1^2+X_1^2} \tag{3.14}$$

with. $|X_1| = \frac{R_1}{Q}$; $|X_2| = QR_2$, unde $Q = \sqrt{\frac{R_1-R_2}{R_2}}$

Note that X_1, the parallel reactance, has a negative reactance because it is usually a capacitor. This gives the L network the additional feature of harmonic suppression, as it is also a low-pass filter. The inverse connection (impedance increase) is simply inverse – for example, series reactance with the source. The size of the impedance ratio is limited by reactance losses, such as inductor Q. Multiple L-sections can be cascaded to achieve higher impedance ratios or higher bandwidth. Transmission line matching networks can be modeled as an infinite number of cascaded L-sections. Optimal adaptation circuits can be designed for a particular system using Smith diagrams.

3.2.4. Power factor correction devices

Power factor correction devices are intended to cancel out the reactive and nonlinear characteristics of a load at the end of a power line. This makes the load seen by the power line purely resistive. For a certain true power required by a load, it minimizes the actual current supplied through the power lines and minimizes the power dissipated (dispersed) in the resistance of those power lines. For example, a maximum power point tracker is used to extract maximum power from a solar panel and efficiently transfer it to batteries, the power grid, or other loads. The maximum power transfer theorem applies to its "upstream" connection to the solar panel, so it emulates a load resistance equal to the resistance of the solar panel's source. However, the maximum power transfer theorem does not apply to its "downstream" connection. That connection is an impedance bridge connection; Emulates a high-voltage, low-resistance source to maximize efficiency.

On the power grid, the total load is usually inductive. Accordingly, power factor correction is most commonly performed with capacitor banks. It is only necessary that the correction be made at a single frequency, the feeding frequency. Complex networks are only needed when a frequency band needs to be right, and this is why simple capacitors are all that is usually needed for power factor correction.

3.3. IMPEDANCE MATCHING USING TRANSMISSION LINES

The impedance bridge is unsuitable for radio frequency (RF) circuits (connections) because it causes power to be reflected back to the source from the boundary between high and low impedance. Reflection creates a standing wave if there is reflection at both ends of the transmission line, which leads to additional energy loss and can cause frequency-dependent losses. In these systems, impedance adaptation is desirable.

In electrical systems involving transmission lines (such as radio and fiber optics) – where the length of the line is large compared to the wavelength of the signal (the signal changes rapidly compared to the time it takes to travel from source to load (load)) – the impedances at each end of the line must be matched with the characteristic impedance of the transmission line to prevent signal reflections at the ends of the line. (When the line length is short compared to the wavelength, impedance mismatch is the basis of transmission line impedance transformers). In radio frequency (RF) systems, a common value for source and load impedance is 50 ohms. A typical RF payload is a quarter-wave antenna with the ground plane (37 ohms with an ideal ground plane); It can be adapted to 50 ohms using a modified ground plane or a matching (matching) coaxial section, i.e. part or all of the higher impedance power supply. Figure 3.6 shows the structure of a coaxial transmission line with a source and a load.

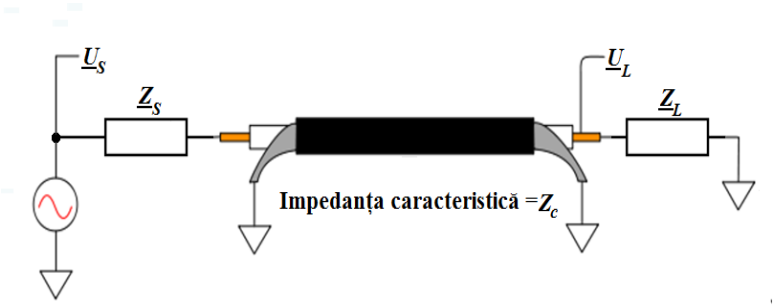


Figure 3.6. Coaxial line with one source and one load

The general form of the voltage reflection coefficient for a wave moving from medium 1 to medium 2 is given by the relationship:

$$\underline{\Gamma}_{12} = \frac{\underline{Z}_2 - \underline{Z}_1}{\underline{Z}_2 + \underline{Z}_1} \tag{3.15}$$

while the voltage reflection coefficient for a wave moving from medium 2 to medium 1 is:

$$\underline{\Gamma}_{21} = \frac{\underline{Z}_1 - \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} \quad \underline{\Gamma}_{21} = -\underline{\Gamma}_{12} \tag{3.16}$$

So, the reflection coefficient is the same (except for the sign) no matter from which direction the wave is approaching the boundary.

There is also a current reflection coefficient, which is the negative of the voltage reflection coefficient. If the wave encounters an opening (a gap) at the end of the load, the positive voltage and negative current pulses are transmitted back to the source (negative

current means that the current is going in the opposite direction). Thus, at each boundary there are four reflection coefficients (voltage and current on one side, and voltage and current on the other side). All four are the same, except that two are positive and two are negative. The voltage reflection coefficient and the current reflection coefficient on the same side have opposite signs. The coefficients of reflection of the voltage on opposite sides of the boundary have opposite signs.

Since they are all the same, except for the sign, it is traditional to interpret the reflection coefficient as the coefficient of reflection of the voltage (unless otherwise indicated). Either end (or both ends) of a transmission line can be a source or a load (or both), so there is no inherent preference for which side of the boundary is medium 1 and which side is medium 2. With a single transmission line, it is customary to define the voltage reflection coefficient for a wave incident at the boundary on the transmission line side, regardless of whether a source or load is connected on the other side.

Load matching is the matching between the characteristic impedance of the line (Z_c = Z0) and the impedance of the load (Z_L). To achieve the two objectives of the adaptation, it must be achieved using only components that do not consume active power.

The objectives of adaptation are:

- Maximizing the power transmitted to the load;
- Minimization of reflected wave.

3.3.1. Adapting a purely resistive load using a transmission line

The simplest case is when the load has a purely resistive impedance. The adaptation in this case can be easily achieved using a transmission line of quarter wavelength and carefully chosen characteristic impedance $\underline{Z'_0}$ (Figure 3.7).

$$\underline{Z}_L = \underline{Z}_0 \tag{3.17}$$

The first step is to calculate the length of the adaptation transmission line [1]. For this, the formula (3.18) is used.

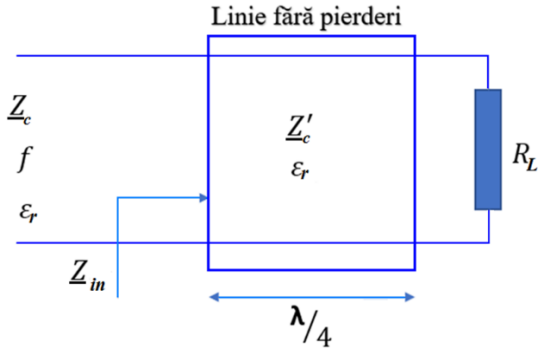


Figure 3.7. Adaptation of a purely resistive load using a transmission line of λ/4 length, [1]

$$\lambda = \frac{c_0}{\sqrt{\varepsilon_r} f} \tag{3.18}$$

To calculate the value of the impedance of the adaptation line, we start from the adaptation condition (3.19) which can also be written according to the reflection coefficient

(3.20). We then recall the formula of the input impedance in a transmission line as a function of the length of the line (3.21). Making in this formula the equality l=λ/4, we obtain the relation (3.21), [1].

$$\underline{Z}_{in} = Z_c = Z_0 \tag{3.19}$$

$$\underline{\Gamma} = \frac{\underline{Z}_{in} - Z_0}{\underline{Z}_{in} + Z_0} = 0 \tag{3.20}$$

$$\underline{Z}_{in} = \frac{Z'_0 + jZ'_0 tg(\beta l)}{Z'_0 + j\underline{Z}_L tg(\beta l)} \tag{3.21}$$

$$\underline{Z}_{in} = \frac{Z'^2_0}{\underline{Z}_L} \tag{3.22}$$

From the relation (3.22) the value of the characteristic impedance of the line that satisfies the adaptation condition is obtained.

3.3.2. Adapting a complex load using passive elements with concentrated parameters (coils or capacitors)

A more complicated case is that of microwave circuits terminated on a load of complex impedance (3.23) that has, in addition to a resistive component, also a reactive one. In this case, the adaptation can no longer be done directly using a single transmission line with a predefined length [1-3].

$$\underline{Z}_L = R_L = jX_L \tag{3.23}$$

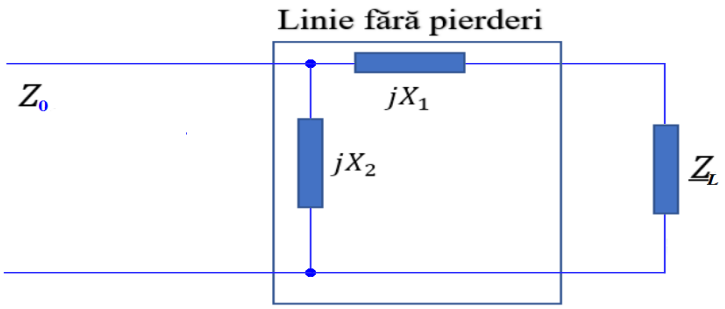


Figure 3.8. Lossless transmission line terminated on any load of $\underline{ZL}$ impedance adapted using two purely reactive components

In this case, the simplest way to adapt is to use two purely reactive components (Figure 3.8), one of them in series with the load and the other in parallel. Depending on the order of the components (read from the load) we can have the case of the parallel-series circuit (the first component is parallel to the load, and the second in series), represented in figure 3.9, and the case of the series-parallel circuit (in which the first component is in series with the load, and the second in parallel) represented in figure 3.10, [1-5].

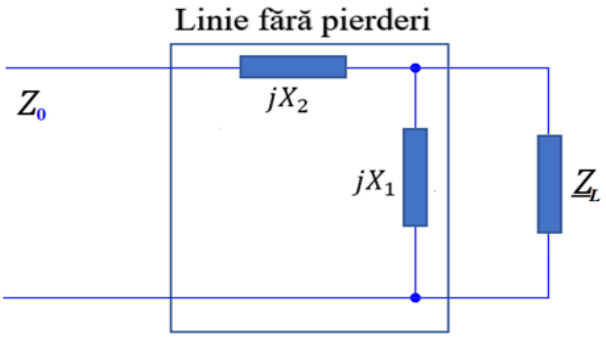


Figure 3.9. Impedance matching using the parallel-series matching circuit

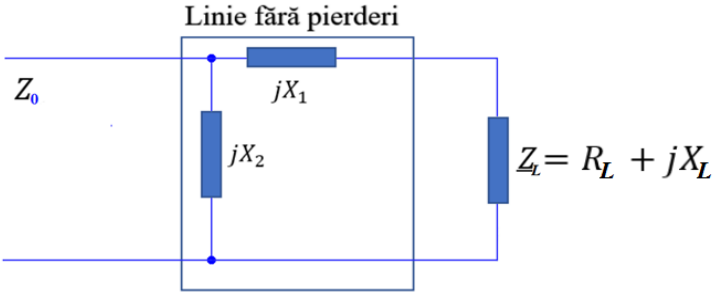


Figure 3.10. Impedance matching using the series-parallel matching circuit

We will analyze the case of the series-parallel circuit below. For this circuit, the equivalent impedance of the load + matching circuit assembly has the form (3.24). The adaptation condition is expressed in this case as an equality between the equivalent and characteristic impedance of the line. After processing the adaptation condition we obtain equality (3.25) which, separated into its real and imaginary parts, we obtain the system (3.26), [1].

$$\underline{Z}_{ech} = \frac{[R_L + j(X_1 + X_L)]jX_2}{R_L + j(X_1 + X_2 + X_L)} = Z_0 \tag{3.24}$$

$$jR_LX_2 - X_1X_2 - X_LX_2 = R_LZ_0 + j(X_1 + X_2 + X_L) \tag{3.25}$$

$$\begin{cases} (R_LX_2 = Z_0X_1 + Z_0X_2 + Z_0X_L)/Z_0^2 \\ (R_LZ_0 + X_1X_2 + X_LX_2 = 0)/Z_0^2 \end{cases} \tag{3.26}$$

Using the characteristic impedance of the line *Z0*, we norm the system of equations above and write the equation (3.26) again, obtaining the equation (3.27):

$$\begin{cases} r_Lx_2 = x_1 + x_2 + x_L \Rightarrow x_2 = \frac{x_1 + x_L}{r_L - 1} \\ r_L + x_1x_2 + x_Lx_2 = 0 \end{cases} \tag{3.27}$$

From the first equation (3.27) we remove X_2 and replace it in the second equation, obtaining equation (3.28) of degree 2, with the final solution for X_1 given in relation (3.29). Substituting equation (3.29) back into equation (3.27) we find the value for X 1 as well. Note that we have two different system solutions (3.27), which result in two possible series-parallel adaptation circuits. The same would have been achieved for parallel-series circuits,

raising the total number of possible circuits of adaptation with elements with parameters concentrated to 4, [1].

$$r_L + \frac{x_1 + x_L}{r_L - 1}(x_1 + x_L) = 0 \Rightarrow \frac{(x_1 + x_L)^2}{r_L - 1} = -r_L \Rightarrow (x_1 + x_L)^2 = r_L(1 - r_L) \Rightarrow$$
$$\Rightarrow x_1 + x_2 = \pm\sqrt{r_L(1 - r_L)} \tag{3.28}$$

$$x_1 = -x_2 \pm \sqrt{r_L(1 - r_L)} \tag{3.29}$$

and, according to the first equation in relation (23), *x2* has the expression:

$$x_2 = \frac{x_1 + x_L}{r_L - 1} \tag{3.30}$$

In a transmission line, a wave travels from the source along the line. Suppose the wave hits a boundary (a sudden change in impedance). Part of the wave is reflected back, while others continue to move further. We assume that there is only one limit, to pregnancy.

Daughter:

- $\underline{Vi}$ and $\underline{Ii}$ the voltage and current entering the limit from the source;
- $\underline{Vt}$ and $\underline{It}$ voltage and current that are transmitted to the load;
- $\underline{Vr}$ and $\underline{Ir}$ voltage and current that are reflected back to the source.

On the line side of the boundary $\underline{Vi} = \underline{ZcIi}$ and $\underline{Vr} = \underline{ZcIr}$ and on the charge side $\underline{Vt} = \underline{ZcIt}$, where $\underline{Vi}$, $\underline{Vr}$, $\underline{Vt}$, $\underline{Ii}$, $\underline{Ir}$, $\underline{It}$, and $\underline{Zc}$ are phasors.

Therefore, at a limit, the voltage and current must be continuous.

$$\underline{V_t} = \underline{V_i} + \underline{V_r}\ ,\ \underline{I_t} = \underline{I_i} + \underline{I_r} \tag{3.40}$$

All these conditions are met by:

$$\underline{V_r} = \underline{\Gamma_{TL}}\underline{V_i},\ \underline{I_r} = -\underline{\Gamma_{TL}}\underline{I_i},$$
$$\underline{V_t} = \left(1 + \underline{\Gamma_{TL}}\right)\underline{V_i},\quad \underline{I_t} = \left(1 - \underline{\Gamma_{TL}}\right)\underline{I_i} \tag{3.41}$$

Where:

$$\underline{\Gamma_{TL}} = \frac{\underline{Z_L} - \underline{Z_c}}{\underline{Z_L} + \underline{Z_c}} \tag{3.42}$$

The purpose of a transmission line is to get the maximum amount of energy to the other end of the line (or to transmit information with minimal error) so that the reflection is as small as possible. This is achieved by adapting the impedants *ZL* and *Zc* so that they are equal ($\underline{\Gamma} = 0$).

At the source end of the transmission line, there may be incident waves from both the source and the line; A reflection coefficient can be calculated for each direction:

$$-\ \underline{\Gamma_{ST}} = \underline{\Gamma_{TS}} = \frac{\underline{Z_S} - \underline{Z_c}}{\underline{Z_S} + \underline{Z_c}} = \underline{\Gamma_S} \tag{3.43}$$

where *Zs* is the impedance of the source.

The source of the incident waves in the line are the reflections at the end of the load. If the impedance of the source matches the line, the reflections from the end of the load will be absorbed at the end of the source. If the transmission line is not adapted to both ends, the reflections from the load will be re-reflected at the source and re-reflected at the end of the load at infinity, losing energy on each transit of the transmission line. This can cause a resonant state and strong frequency-dependent behavior. In a narrowband system, this may be desirable for adaptation, but is generally undesirable in a broadband system.

The impedance at the line input has the expression:

$$\underline{Z}_{in} = \underline{Z}_c \frac{(1+\underline{T}^2\underline{\Gamma}_L)}{(1-\underline{T}^2\underline{\Gamma}_L)} \tag{3.44}$$

where *T* is the unidirectional (end-to-end) transfer function when the transmission line is exactly matched to the source and load. *T* takes into account everything that happens with the signal in transit (including delay, attenuation, and dispersion). If there is a perfect adaptation to the load, and the source can calculate the voltage at the terminals.

The voltage at the load terminals has the expression:

$$\underline{V}_L = \underline{V}_S \frac{\underline{T}(1-\underline{\Gamma}_S)(1+\underline{\Gamma}_L)}{2(1-\underline{T}^2\underline{\Gamma}_S\underline{\Gamma}_L)} \tag{3.45}$$

where *VS* is the open-circuit (or discharged) output voltage from the source. Note that if there is a perfect fit at both ends $\underline{\Gamma}_L = 0$ and $\underline{\Gamma}_S = 0$ then:

$$\underline{V}_L = \underline{V}_S \frac{\underline{T}}{2} \tag{3.46}$$

3.4. USE OF THE SMITH DIAGRAM IN THE ADAPTATION OF IMPEDANCES

In order for there to be an optimal transfer of power between the generator and the load, the conjugate adaptation condition must be met , [1]:

$$\underline{Z}_{in} = \underline{Z}_G^* \tag{3.47}$$

When the generator is adapted to the transmission line ($\underline{Z}_G = Z_0$) condition (3.47) is reduced to the condition that the load is also *adapted to the transmission line*:

$$\underline{Z}_L = Z_0 \tag{3.48}$$

When the condition (3.48) of adaptation is met, then we have no reflected wave from the load. A measure of adaptation can be given by the *RL* (*return loss*) parameter which measures the attenuation of the signal reflected by the load, [1]:

$$R_L(dB) = 20\, log_{10} \frac{P_r}{P_i} \tag{3.49}$$

where *Pi* is the power of the wave incident on the load and *Pr* is the power of the wave reflected by the load.

In the ideal case of a perfectly matched load, the RL parameter tends towards -∞. Taking into account that the powers of the forward and reverse waves are proportional to the square of the amplitudes, then we can write the RL parameter and depending on the reflection coefficient $\underline{\Gamma}$, [1]:

$$R_L(dB) = 20\, log_{10} \frac{|\underline{U}^-|}{|\underline{U}^+|} = 20\, log_{10} |\underline{\Gamma}| \tag{3.50}$$

We can introduce a new parameter that gives us an idea of how selective the adaptation is, in other words how large the spectral range over which the adaptation is made. We will set the limit of the mismatch at -30dB, i.e. we will consider that the adaptation is achieved if, [1]:

$$|R_L(dB)| > 30 \text{ dB} \tag{3.51}$$

We will consider the frequency band (*B*) around the center frequency *f0,* i.e. the difference between the maximum frequency (*f2*) and minimum frequency (*f1*) for which the modulus of the parameter *RL* is greater than 30 dB. We can then define the quality factor of the adaptation circuit as, [1]:

$$Q = \frac{f_0}{B} = \frac{f_2 + f_1}{2(f_2 - f_1)} \tag{3.52}$$

3.3. CONCLUSIONS

Impedance matching is the practice of designing the input impedance of an electrical load or the output impedance of the corresponding signal source to maximize power transfer or minimize signal reflection from the load. It is widely used in: analog electrical circuits, wireless power transfer systems, transmission lines, electrical circuits and RF transmission lines, etc. This chapter gives a correct definition of impedance matching in electrical engineering. The design and construction of the impedance adaptation are extensively presented by calculating with great accuracy the values of the parameters (coils and capacitors) used in the construction of the adaptation circuits. An electrical power source, such as a generator, amplifier, or radio transmitter, has a source impedance equivalent to an electrical resistance in series with a frequency-dependent reactance. Likewise, an electrical charge, such as a light bulb, transmission line, or antenna, has an impedance equivalent to a resistance in series with a reactance.

Almost all impedance matching procedures are based on the maximum power transfer theorem. This theorem states that the maximum active power is transferred from the source to the load when the load resistance is equal to the resistance of the source and the load reactance is equal (with the negative of the source reactance) to the reactance of the source with changed sign: the reactances cancel each other out with their opposite frequency dependence. In a direct current (dc) circuit, the condition is met if the load resistance is equal to the resistance of the source. In an alternating current (AC) circuit, the reactance depends on the frequency, so circuits that are impedance-matched to a frequency may not be impedance-matched if the frequency is changed. It is shown that matching impedance over a wide frequency band will generally require complex, filter-like structures with many components, except in the mundane case of constant source and load resistances, when a transformer can be used.

The chapter introduces a maximum power point tracker used to extract maximum power from a solar panel and efficiently transfer it to batteries, the power grid, or other loads. The maximum power transfer theorem applies to its "upstream" connection to the solar panel, so it emulates a load resistance equal to the resistance of the solar panel's source. However, the maximum power transfer theorem does not apply to its "downstream" connection. That connection is an impedance bridge connection; Emulates a high-voltage, low-resistance source to maximize efficiency.

Since on a power grid the total load is usually inductive, power factor correction is most commonly performed with capacitor banks. It is only necessary that the correction be made at a single frequency, the feeding frequency. Complex networks are only needed when a frequency band needs to be right, and this is why simple capacitors are all that is usually needed for power factor correction.

The PR - Return Loss parameter, which measures the attenuation of the signal reflected by the load and the IL - Insertion Loss parameter are effective measures to measure adaptation.

Using the S parameters, the performance of a radio frequency (RF) circuit can be fully characterized without the need to know the internal composition. S-parameters are used with great success in determining the performance of impedance matching in wireless power transfer systems.

Through numerous examples of the use of the Smith diagram in determining the circuit elements used in impedance adaptation, it confirms the outstanding efficiency of this diagram in the design of impedance adaptation schemes.

4. DESIGNING IMPEDANCE MATCHING NETWORKS

Impedance matching networks can consist only of concentrated circuit elements (coils (inductors), capacitors, resistors), transmission lines, or a combination of concentrated circuit elements and transmission lines. Here we will consider only coils (inductors) and capacitors as concentrated circuit elements, although resistors (resistors) can also be used, especially for broadband frequency adaptation of impedance adaptation, [1].

Impedance matching circuits (Networks) with concentrated circuit elements or transmission lines can be made independently or from each other. The use of concentrated circuit elements, for the realization of impedance matching circuits (networks), is usually preferred over transmission lines to radio frequency integrated circuits (ICFR – RFIC), due to their significantly smaller dimensions. However, at high frequencies, at the higher end of the radio frequency (RF) spectrum, transmission line impedance matching networks should be used because the concentrated circuit elements do not truly behave as concentrated elements due to the high frequencies and/or size of the transmission lines, and become comparable to those of the concentrated elements while the transmission lines have a higher quality factor.

4.1. TOPOLOGIES OF IMPEDANCE MATCHING CIRCUITS (NETWORKS)

Figure 4.1 shows eight possible topologies for impedance matching circuits (networks), taking into account only two connected (aggregated) elements. These impedance matching circuits (networks) are the simplest practical circuits (networks) for matching circuit elements to concentrated parameters.

An inductor and a capacitor are equivalent to a lossless (open) or short-circuited transmission line. Therefore, impedance matching networks with concentrated circuit

elements in Figure 4.1 can be transferred to equivalent impedance matching (matching) networks using open and/or short-circuited transmission lines. Figure 4.2 shows two of them corresponding to the impedance matching networks of the concentrated element in Figure 4.1, (a) and (c) where short-circuit and no-load (open-circuit) transmission lines (series/shunt) for inductor (series/shunt), and capacitor, respectively, are used. It is observed that series transmission lines are very difficult, and even impossible to achieve for certain transmission line configurations, in radio-frequency integrated circuits (ICFR – RFIC). To avoid the problem of serial connection, Kuroda identities can be used to transform the serial configuration into transmission lines in parallel connection (shunt), [1-3].

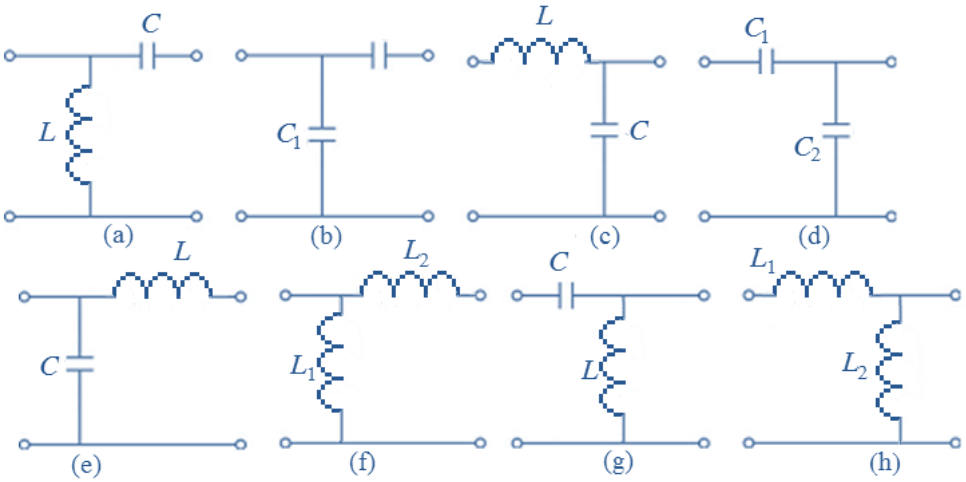
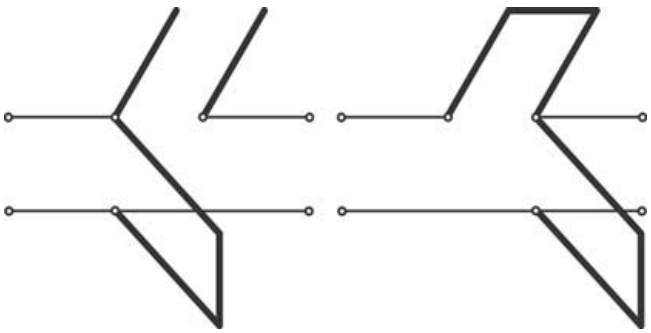


Figure 4.1. (a-h). Circuits (networks) possible to adapt the immedance, using two circuit elements with concentrated parameters.



a) b)

Figure 4.2. Impedance matching circuits (networks) for the transmission lines corresponding to Figures 4.1, (a) and (c).

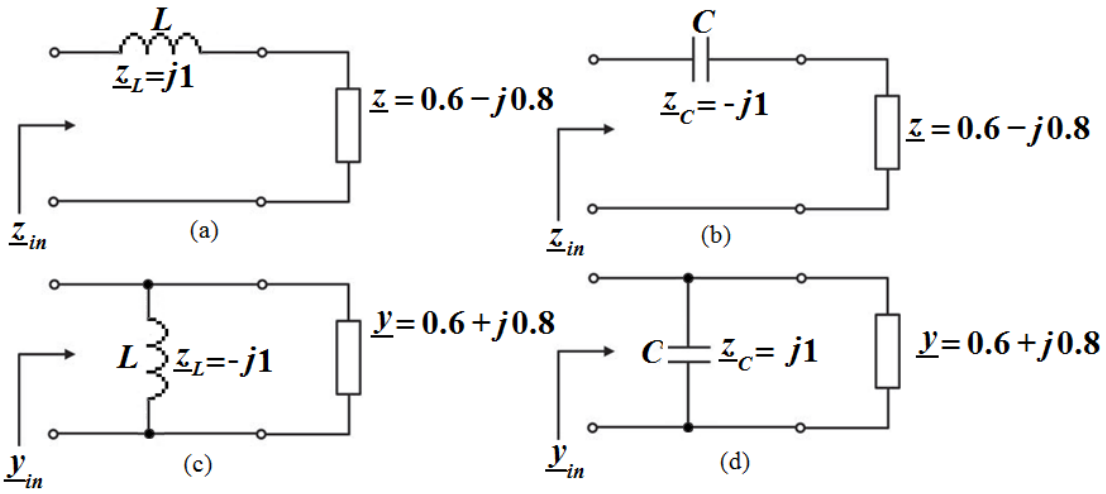


Figure 4.3 a–d. Series or lead (shunt) impedance with coil (inductor) or capacitor. $\underline{z} = 0.6 - j0.8$ and $\underline{y} = 1/z = 0.6 + j0.8$ are the normalized impedance and admittance of a coil (of an inductor). $\underline{zL} = j1$ and $\underline{yL} = 1/zL = -j1$ are the normalized impedance and admittance of a coil (of an inductor), respectively. $\underline{zC} = -j1$ and $\underline{yC} = 1/\underline{zC}$ are the normalized impedance and admittance of a capacitor, respectively.

4.1.1. Effect of inductor (coil) series

An impedance connected in series with an inductor (with a coil) is considered, as shown in Figure 4.3, (a). The normalized input impedance representing the transformed impedance is $\underline{Zin} = \underline{ZL} + \underline{Z} = 0.6 + j0.2$. The impedance $\underline{Z}$ and its transformed impedance Zin are located at points A and B on a Smith *diagram* $\underline{Z} - \underline{Y}$, respectively, as shown in figure 4.4.

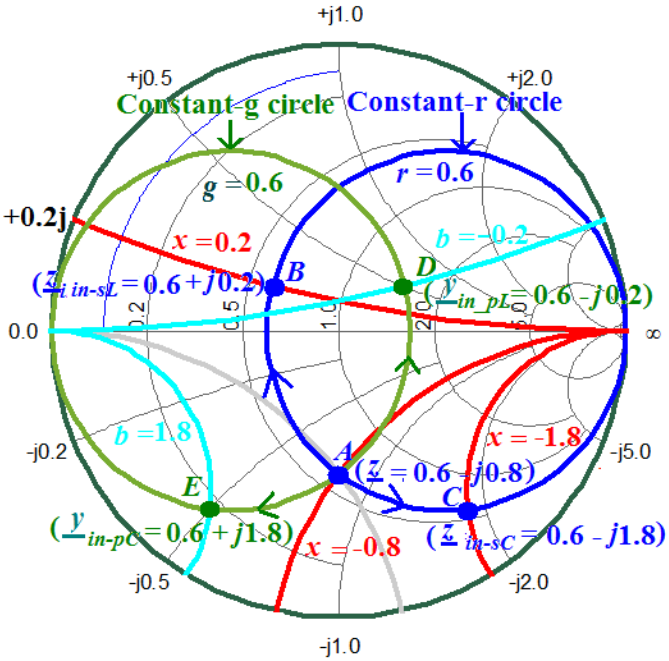


Figure 4.4 Impedance transformation by an inductor or by a capacitor on the Z – Y Smith diagram.

It should be noted that these points are located according to the Z diagram, portion of the Smith Z – *Y* diagram. Then we can see that a series inductor transfers an impedance (A) to another impedance (B) on the same r-constant circle clockwise (CW). The r-constant circle refers to the *Z portion* of the Smith chart.

4.1.2. Series Capacitor Effect

Now consider figure 4.3, b showing the same impedance *z* as shown in figure 4.3, a, in series with a capacitor. The transformed impedance is $\underline{zin} = \underline{zC} + \underline{z} = 0.6 - j1.8$ and is located at point C in Figure 4.4. It is obvious that a series capacitor converts one impedance (A) into another impedance (C) on the same r-constant circle in a counterclockwise direction (CCW).

4.1.3. Effect of the shunt inductor (coil) (parallel, shunt)

Figure 4.3, c shows an admittance y in parallel with an inductor. The transformed admittance is $\underline{yin} = \underline{yL} + \underline{y} = 0.6 - j0.2$ and is located at point D in Figure 4.4. Note that D is located according to the Y plot portion of the Smith *Z – Y chart*. The result indicates that a shunt inductor moves an admittance (A) to another admittance (D) located on the same g-constant circle in the direction of CCW. The g-constant circle refers to the Y portion of the Smith diagram.

4.1.4. Effect of the shunt capacitor (shunt)

Figure 4.3, d shows the same y-admittance in parallel with a capacitor. The transformed admittance is $\underline{yin} = \underline{yC} + \underline{y} = 0.6 + j1.8$ and is located at point E in figure 4.4. We can see that a shunt capacitor causes an admittance (A) to move to another admittance (E) on the same g-constant circle clockwise (CW).

Table 4.1 Summary of transformation effects of coils and capacitors on impedance or admittance

Circuit Element	The direction of transformation	The Circle of Transformation
Series Coil	Clockwise (SAC)	*r* – constant (*Diagram Z*)
Series Capacitor	Counterclockwise (SIAC)	*r* – constant (*Diagram Z*)
Shunt coil (shunt)	Clockwise (SAC)	*g* – constant (Y-Diagram)
The shunt condesor (derivation)	Counterclockwise (SIAC)	*g* – constant (Y-Diagram)

Table 4.1 and Figure 4.5 summarize the transformation effects of inductors and capacitors on impedance (or admittance). It is noted that Table 4.1 or Figure 4.5 can also be used to identify whether a series or shunt coil or capacitor is required to move an impedance (or an admittance) on an r-constant or g-constant circle. They can be used to design virtually any impedance matching circuit with concentrated circuit elements.

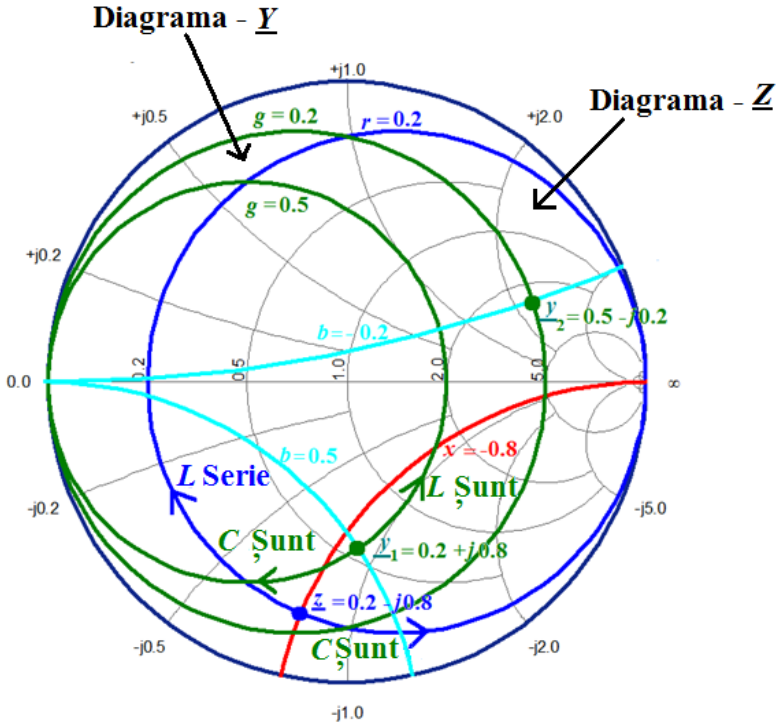


Figure 4.5. Effects of coil and capacitor transformation on impedance or admittance on the Smith Z - *Y diagram*.

In practice, however, not all designed matching networks can be made from obtained elements such as inductors or capacitors may not be feasible. This, in turn, limits the number of possibilities for impedance matching networks. In addition, the choice of a particular impedance matching network depends on the number of elements used, in which fewer elements are usually preferred due to the possibility of losses and smaller sizes, the quality factor of one element over another (such as inductor versus capacitor), and the desired frequency response, such as low-pass response, pass up or pass lane. It is noted that although only two elements are used for impedance matching networks in the above examples, the procedure can be easily extended for impedance matching networks that have more than two elements.

4.3. TRANSMISSION LINE IMPEDANCE MATCHING CIRCUITS (NETWORKS)

Transmission line matching networks are inevitable in the high frequency domains of RF circuits, where concentrated elements are not feasible and/or have dimensions comparable to a lower quality factor with transmission lines. In general, transmission line matching networks consist of open (empty) and short-circuited series and shunt transmission lines and

cascade transmission lines. Different types of transmission lines (e.g. microstrip line or coplanar waveguide) or a combination of different transmission lines can be (or may be required) used to achieve a certain impedance with high or low characteristic, a certain transmission line configuration (e.g. series connection or short-circuited), a specific interface or connection with other circuit elements or circuits.

4.3.1. Quarter-wave transformer

The quarter-wave transformer is a transmission line that has a physical length of λ/4, where λ is the wavelength or an electrical length of 90º, used to adapt two real impedances. We consider a lossless quarter-wave transformer connected to a real impedance at each end like the one shown in Figure 4.8.

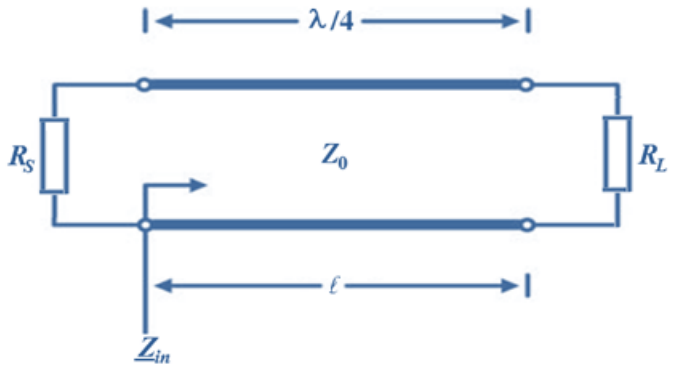


Figure 4.8 Quarter-wave transformer

The input impedance of the quarter-wave transformer is obtained with the formula:

$$\underline{Z}_{in} = Z_0 \frac{R_L + jZ_0 \tan(\beta l)}{Z_0 + jR_L \tan(\beta l)} = \frac{Z_0^2}{R_L} \tag{4.1}$$

Using βl = π/2, equation (4.1) leads to equation 4.2, after applying the perfect match condition of the impedance $\underline{Zin} = Rs$:

$$Z_0 = \sqrt{R_L R_S} \tag{4.2}$$

specifying the characteristic impedance of a quarter-wave transformer that ensures impedance matching between two real impedances.

If one of the impedances is not real, for example, the ZL load impedance, then we can convert *ZL* to a real *RL impedance* by inserting a transmission line immediately in front of *ZL* before using a quarter-wave transformer to match *RL* to RS, as shown in Figure 4.9, (a). Suppose the inserted transmission line is lossless, the displacement along the transmission line is equivalent to the displacement on an *r*–constant circle on a Smith diagram. Therefore, the complex impedance $\underline{ZL}$ can be converted to a real *impedance RL* by moving towards the generator (clockwise direction (CW)) along the circle *r* – constant to *ZL* on a Smith diagram, as shown in figure 4.9 (b). There are two real values for *RL* which result in two possible

values for the length of the transmission line (*d1* and *d2*) and which can be determined directly from the Smith diagram, as is done in Figure 4.9, b. The characteristic impedance *Z0* of the quarter-wave transformer can then be determined using equation (4.2). Knowing *Z0*, *Rs*, *d1* (or *d2*) and *ZL* allows the determination of the characteristic impedance Z'_0 of the introduced transmission line.

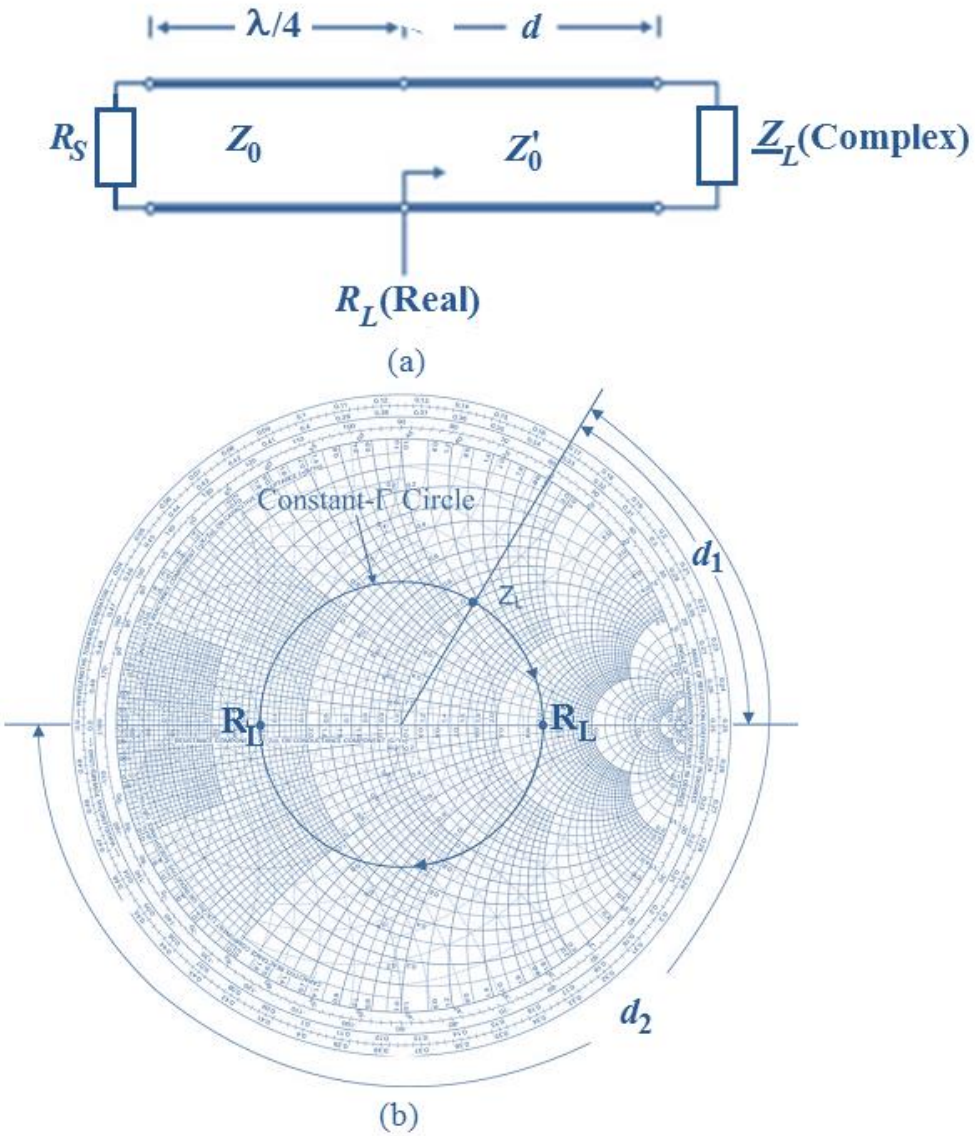


Figure 4.9 (a), (b) Adaptation between a complex impedance and a real impedance using a spherical wave transformer and a transmission line.

4.3.2. Single-End Impedance Matching Networks

Figure 4.10 shows four circuits (networks) impedance matching lines for transmission lines, namely single-block impedance matching networks, consisting of a shunt connection transmission line, open series or short circuit (end - stub) and a cascade transmission line. Series stub-based matching networks are difficult to achieve unless serial stubs are converted to shunt stubs using techniques such as Kuroda identities. These transmission line matching

networks are the basic matching networks, and those that use shunt elements are well-known to RF circuit engineers. These matching networks, as well as their design, serve as a basis that can be expanded to develop and design other topologies for transmission line matching networks.

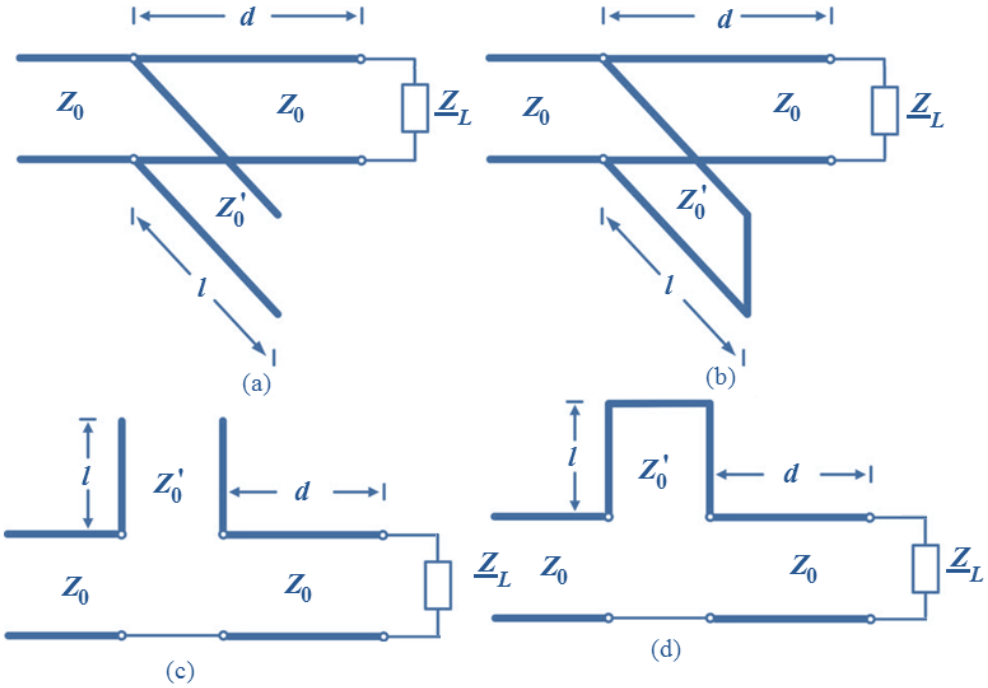


Figure 4.10. (a–d) Single-End Impedance Matching Circuits

We illustrate the formulation of the project (design) by considering a single-ended network (stub), using a short-circuited stub (end) such as the one shown in Figure 4.11, which matches a complex load impedance $\underline{ZL}$ with a real impedance *Z0* representing the characteristic impedance of the transmission line connected to the input of the matching network or to the impedance presented at that input port. All impedances and admittances are normalized with respect to the characteristic impedance *Z0* and the characteristic admittance *Y0* = 1/Z0, respectively. We define $\underline{yM}$ as the normalized admittance that looks into the matching network, $\underline{yS}$ as the normalized admittance that looks into the shorted tube (end), $\underline{yT}$ as the normalized admittance that looks into the cascade transmission line, and $\underline{yL}$ and $\underline{zL}$ as the admittance and impedance of the normalized load, respectively.

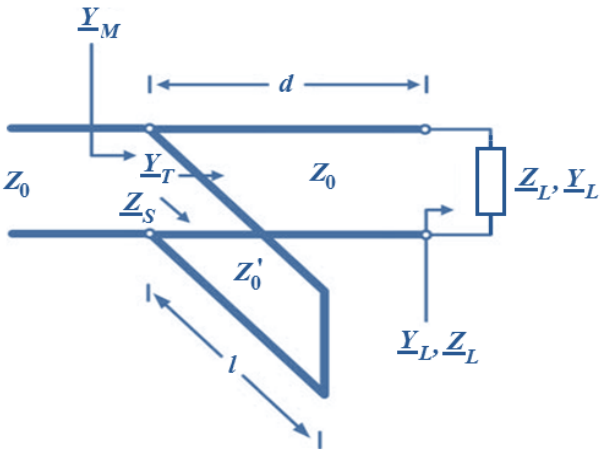


Figure 4.11. Short-circuited single-end (stub) adapter circuit.

Note that in order for a perfect match to occur between $\underline{Y}_L$ and Y_0, the admittance that looks at (looks) the adaptive network match must be equal to Y_0 or:

$$\underline{y}_M = \underline{y}_T + \underline{y}_S = 1 \tag{4.3}$$

Equation (4.3) sets out the criterion from which the lengths of transmission lines in the matching network can be determined for the given characteristic impedances. This criterion can be rewritten under the following two conditions when using the normalized input admit of the shorted stub (end) $\underline{y}_S = -\mathrm{j}Y'_0$ elbow $\left(\beta l / Y_0\right)$:

$$\underline{y}_T = 1 + jbS, \tag{4.4}$$

$$\underline{Y}_S = -jbS. \tag{4.5}$$

Equation (4.4) establishes the criterion for determining the length d; it shows that d is determined such that the real part of $\underline{yT}$ is equal to 1. Equation (4.5) shows that the length l is determined such that $\underline{yS} = -jbS$. Note that the shorted stub (end) in Figure 4.11 can be replaced with an open-circuit stub (end) whose admittance is $\underline{y}_S = jY'\mathrm{o} \tan \beta \mathrm{l}/\mathrm{Y}_0 \equiv -jbS$. The same design formulation also applies to single-stub (end) matching networks that use open-circuit stubs (ends) and can be extended to multi-stub impedance matching networks (ends). Figure 4.12 shows some possible double-stub (double-end) impedance matching networks.

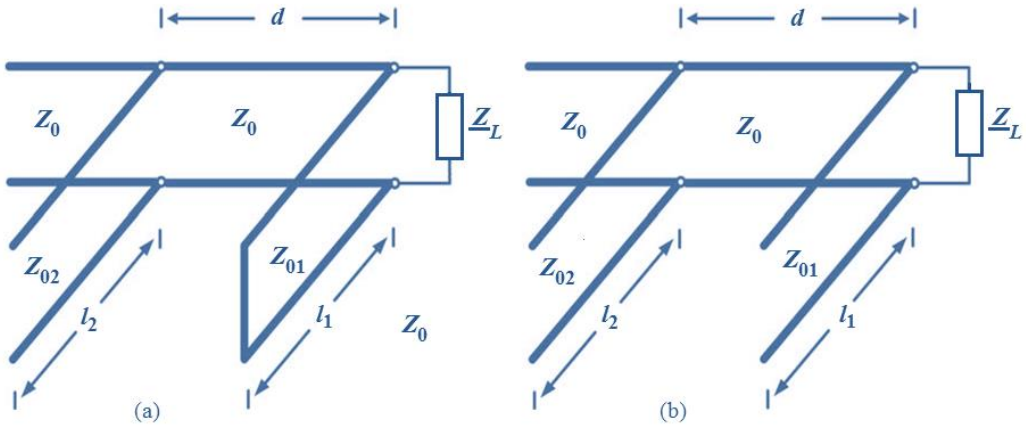


Figure 4.12. (a), (b) Double-ended impedance matching circuits

4.4. CONCLUSIONS

The design of impedance matching networks is an essential process in radio frequency (RF) engineering, telecommunications, and microwave circuits, having a direct impact on the efficiency of power transmission between source and load.

The study highlighted the fact that the design of impedance matching networks is an essential step in the realization of modern electronic and communication systems, ensuring optimal power transfer and reduction of reflections on transmission lines.

The use of the Smith diagram demonstrates the efficiency of graphical and analytical approaches in the design of adaptation networks, providing a solid basis for understanding the phenomena of propagation and interaction between source and load.

The results showed that perfect adaptation can be achieved especially on narrow frequency bands, and in current applications a compromise between maximum efficiency and bandwidth expansion is necessary, especially in the context of broadband communication technologies.

At high frequencies, the actual behavior of the components introduces deviations from the theoretical model. Thus, the design of adaptation networks requires both a rigorous theoretical foundation and an increased attention to technological and constructive aspects.

Recent research highlights the transition from discrete implementations to integrated, reconfigurable solutions, where impedance matching is performed directly in RF circuits or through active tuning techniques. These directions reflect the increasingly stringent requirements of modern applications (5G, IoT, satellite communications).

The development of CAD tools and electromagnetic simulation methods has led to an advanced optimization of adaptation networks, allowing the exploration of complex topologies and reducing design time.

5. THEVENIN, NORTON AND HYBRID EQUIVALENT CIRCUIT

5.1 INTRODUCTION

In today's high-speed technology, analog and mixed-signal integrated circuit technology occupies an important and decisive place in communication and signal processing. In particular, with CMOS technology rapidly embracing the field, designing analog circuits has become more difficult than ever [1-11]. Other developments in technology, such as: lower supply voltages, reduced power consumption, complexity and performance, and large number of transistors, have substantially increased the demand for new design methodologies and techniques.

A major difficulty in dealing with analog circuits is bias in dc - obtaining the desired operating (operating) points with rapid convergence; And the problem is getting worse with the advancement of technology, which is due to the increase in the size and complexity of the

circuit. Analysis can even lead to multiple operating points in cc or instability in operating points caused by positive feedbacks [3,12,13]. In the SPICE circuit simulator [1-7], for example, methods such as Newton-Raphson iteration techniques are used to deal with nonlinearities. The major difficulty is sometimes to make the circuit converge in a limited number of iterations. Schemes such as adding minimum conductance (GMIN), shunt resistances, changing tolerance values for results, and supply steps are usually adopted in the simulator to make convergence possible.

Typically, poor selection of initial conditions or the adoption of large, unregulated steps of iterations causes instability or may even cause non-convergence of the computational process. Another difficulty may result from a fixed circuit topology with fixed DC sources during the polarization procedure. With such pre-established conditions, operating points are found naturally through long and timely iterations. All of this adds up to the design task and the process on time. We need a guided design procedure; a procedure that helps a designer go through a top-down and piece-by-piece design strategy.

The objective of this chapter is to introduce such a guided design procedure for polarization. Our strategy separates the linear and nonlinear portions of an analog circuit and takes more control over the nonlinear portions. This separation of portions (components) within the circuit is achieved by introducing a new gate modeling that cancels out the ports of nonlinear devices. This, in turn, leads to a new polarization technique for nonlinear components. The result is the replacement of ordinary DC power supplies with alternative sources that are attached directly to nonlinear devices. It is shown that a unique and very strong additive property is in charge of performing this component polarization operation. Another useful property that uses this strategy is the elimination of nonlinearity in the design of the polarization process. This is achieved because, being locally polarized, the nonlinear components can be replaced by their linear models operating at those points Q; Therefore, the circuit polarization design is completely linear.

Paragraph 5.2 defines the Thévenin and Norton equivalent circuits, specifying the necessary and sufficient conditions for a singleport linear circuit to be substituted by a Thévenin or Norton equivalent circuit. Paragraph 5.3 sets out the necessary and sufficient conditions for a uniport linear circuit to be replaced by an equivalent hybrid circuit.

The Thévenin, Norton, and Hybrid equivalent circuits are used in the construction of Nullified Hybrid equivalent circuits. The equivalent Thévenin, Norton, hybrid and null hybrid circuits are used for the local polarization of analog circuits and for simplifying the analysis of complex nonlinear analog circuits with a reduced number of nonlinearities, [1-15]. Several illustrative examples are presented that highlight the veracity of the elaborated provisions.

5.2. THEVENIN AND NORTON EQUIVALENT CIRCUITS

The Thévenin and Norton equivalent circuits, also known as the Thévenin and Norton models, are two well-known conventional models that explain the terminal behavior of linear circuits. They are crucial for circuit analysis, replacing a terminal port with a source and impedance [1-14]. Both Thévenin and Norton models are very useful circuit simplification

techniques that are often used to focus only on the terminal behavior of certain linear portions of a circuit that normally provide power or signal to the rest of the circuit. In general, both models are used in different circuit analyses and applications, such as, in source transformation, direct current (dc) analysis, sinusoidal and time-domain analysis – using certain transformation techniques (in the frequency or phasor range and in the s-domain) [1-9]. Here we limit our discussion only to continuous circuit analysis.

Definition 1 says that a single-gate *circuit N* is well defined, relative to that gate, if it does not contain any circuit element that is coupled, electrically or non-electrically, to a physical variable outside N: e.g., controlled sources that depend on a variable external to *N*, transformer windings magnetically coupled to an external winding, a photoresistor coupled to an external light source, etc.

Any resistive, linear, time-invariable, well-defined Uni-port N that meets the following unique solvency condition may be replaced by an equivalent uni-port without affecting the solution of any external circuit (not necessarily linear or resistive) connected to the Uni-port *N*.

5.2.1. Thévenin *Neq* equivalent circuit for a uni-port

Thévenin's theorem was first discovered by the German scientist Hermann von Helmholtz in 1853 and was rediscovered in 1883 by the French telegraphy engineer Léon Charles Thévenin (1857–1926).

Figure 5.1, a represents a linear resistive circuit with two terminals, N, with both independent and dependent sources inside.

The single solvency condition for a single-port circuit Thévenin equivalent *Neq*. A uni-port circuit N (fig. 5.1, a) obtained by connecting an ideal independent source of current *j* to terminals A, B of *N* (oriented from node B to node A) has a unique solution for any value of current *j*.

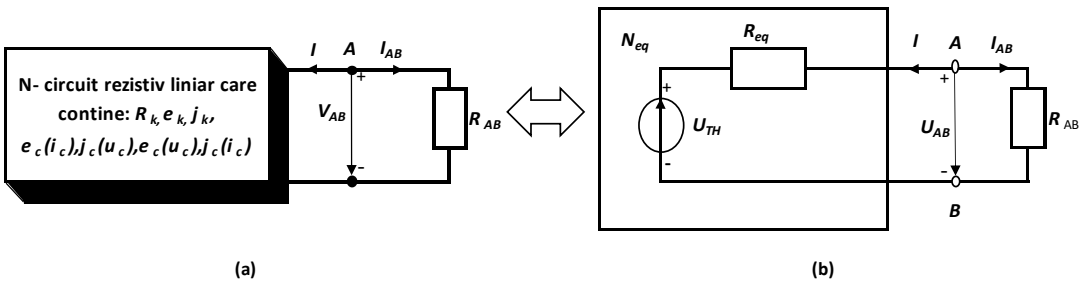


Figure 5.1. a) A linear resistive circuit with two access terminals to which the RAB resistor is connected; b) The Thévenin equivalent circuit.

If the circuit *N* in Figure 5.1 (a) satisfies the single solvency condition above, then the *circuit N* can be substituted with the equivalent voltage generator with $Ee = U_{Th} = UAB0$ and $Ri = Req = RAB0$ - *called the uni-port circuit equivalent Thévenin* N_{eq} (fig. 5.1 (b)). If between terminals A, B the resistor is connected with the *resistance RAB*, then the current *IAB* has the expression:

$$I_{AB} = -I = \frac{U_{Th}}{R_{eq} + R_{AB}}, \tag{5.1}$$

where: $R_{eq} = R_{AB0}$ is the equivalent resistance with respect to terminals A, B (when) of the passivated $R_{AB} = \infty$ *circuit N* (all t.e.m. *e* (all currents *j*) of the independent voltage sources (of current are equal to zero) – called the *equivalent Thévenin resistance*; $U_{Th} = U_{AB0}$ represents the voltage between terminals A and B when idling (when) $R_{AB} = \infty$.

5.2.2. Norton *Neq* equivalent circuit for a uni-port

Norton's theorem was published by engineer Edward Lawry Norton (1898–1983) in 1926 at Bell Labs.

Unique solvency condition for a Norton uni-port circuit equivalent *Neq*: A single-port *circuit N* (fig. 3.2 (a)) obtained by connecting an ideal voltage-independent source *e* to terminals A, B of *N* (oriented from node B to node A) has a unique solution for any value of t.e.m. *e*.

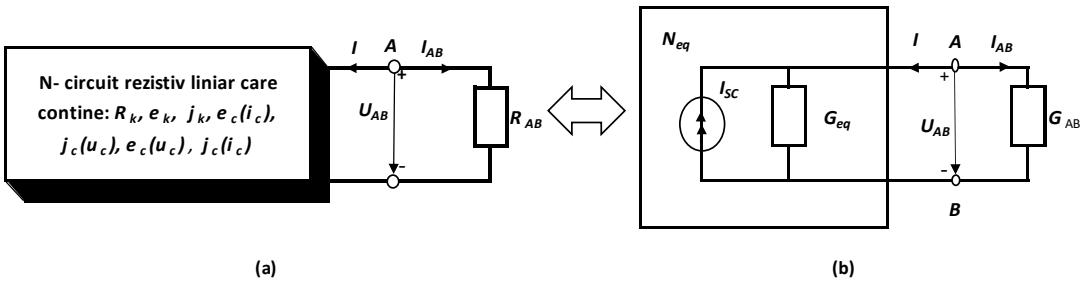


Fig. 5.2. a) A two-port linear resistive circuit; b) Norton equivalent circuit.

When circuit *N* in Figure 5.2(a) satisfies the unique solvency condition corresponding to the Norton equivalent circuit, then circuit *N* may be substituted for the current equivalent generator with $J_e = I_{sc}$ and $G_i = G_{eq}$ *called the Norton N eq equivalent uni-port circuit* (Figure 5.2 (b)). If between terminals A, B the resistor with the conductance GAB, then the *voltage UAB* can be calculated with the formula:

$$U_{AB} = \frac{I_{sc}}{G_{eq} + G_{AB}}, \tag{5.2}$$

where: $G_{eq} = G_{AB0}$ is the conductance equivalent with respect to terminals A, B (when) of the passivated $R_{AB} = \infty$ *circuit N* (all t.e.m. *e* (all currents *j*) of the independent voltage sources (of current are equal to zero) – called *Norton equivalent conductance*; $Isc = I_{ABsc}$ represents *the short-circuit current* between terminals A and B when a resistor $RAB = 0\ \Omega$ is connected between these terminals. Equation (5.2) is called in the literature the current generator theorem or Norton's theorem.

Before proving the above theorems, let's first consider some interpretations and applications of these theorems:

1. The main value of Thévenin's theorem, as well as Norton's theorem, is that it allows us to replace *any* part of a circuit that forms a single *linear resistive uni-*

port (but is of no interest in a given situation) with only two circuit elements, without affecting the solution of the rest of the circuit;

2. Daughter. If we short-circuit the equivalent Thévenin $R_{eq} \neq 0 Neq$ *circuit* and solve the circuit thus obtained in relation to the current *I*, we obtain (see figure 5.1):

$$I_{sc} = -I_{sc} = I_{ABsc} = \frac{U_{Th}}{R_{eq}} \tag{5.3}$$

3. When , the uni-port circuit $R_{eq} \neq 0$ şi $G_{eq} \neq 0 N$ has, according to the equivalence theorem of independent real sources, a Thévenin equivalent circuit as well as a Norton equivalent circuit. Equivalence relations have the expressions:

$$\text{Parametrii Thévenin: } \begin{cases} R_{eq} \\ U_{AB0} = U_{Th} \end{cases} \Rightarrow \text{Parametrii Norton: } \begin{cases} G_{eq} = \dfrac{1}{R_{eq}} \\ I_{sc} = I_{Absc} = \dfrac{U_{Th}}{R_{eq}} \end{cases};$$

$$\text{Parametrii Norton: } \begin{cases} G_{eq} \\ I_{sc} = I_{ABsc} \end{cases} \Rightarrow \text{Parametrii Thévenin: } \begin{cases} R_{eq} = \dfrac{1}{G_{eq}} \\ U_{Th} = \dfrac{I_{sc}}{G_{eq}} \end{cases}. \tag{5.4}$$

4. When, a uni-port *N* is equivalent with respect to two access terminals A, B with both a Thévenin equivalent circuit and a Norton equivalent circuit, *its input characteristic* (*of the operating point*) is defined by the relations:

$$U_{AB} = R_{eq} I + U_{Th} = -R_{eq} I_{AB} + U_{Th} \text{ sau } I = G_{eq} U_{AB} - I_{sc}. \tag{5.5}$$

This feature of the operating (entry) point consists of a *straight line with a slope* G_{eq} and the intersection with the ordinate in the U_{AB} – I *plane* is *Isc*, as shown in fig. 5.3, a, or with a slope Req and the U_{Th} *intersection in the* I-U_{AB} *plane*.

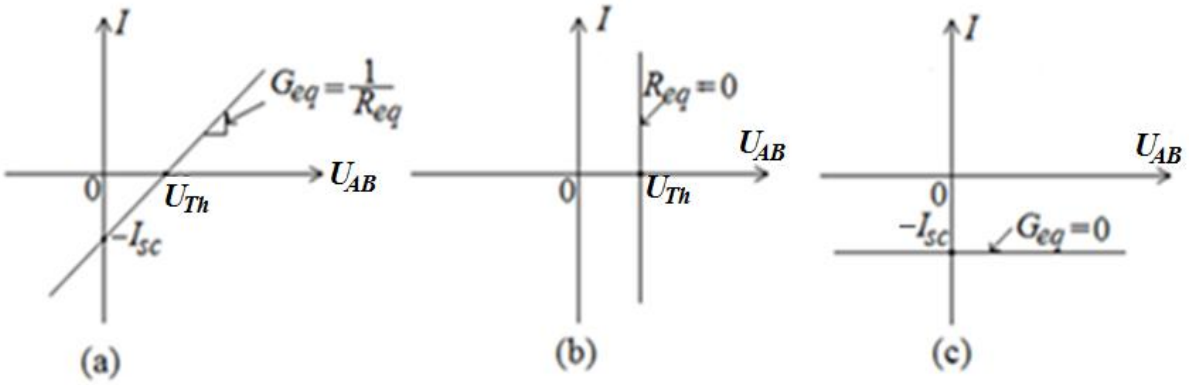


Fig. 5.3. a) The characteristic of the (input) entrainment point of N with $Uos = U_{Th} > 0$ and $Geq > 0$; b) The characteristic of the (input) function point with $UAB = U_{Th}$ and $Req = 0$; c) The characteristic of the (input) function point with $I = -I_{sc}$ and $Geq = 0$.

The function point characteristic in Figure 5.3(a) is drawn for the case where $Geq > 0$, $Uoc = U_{Th} > 0$ and $Isc > 0$.

5. The limiting case $Req = 0$ is shown in Figure 5.3 (b). A Thévenin equivalent circuit in this case consists only of an ideal independent source of voltage battery $Uoc=UTh$ volts. A corresponding Norton equivalent circuit does not exist because $Geq = \infty$. Indeed, the unique solvency condition fails in this case – Kirchhoff theorem two (TIIK) is not valid when a voltage source is applied. The "dual" limit case $Geq = 0$ is shown in Figure 5.3 (c). A Thévenin equivalent circuit does not exist in this case, while the Norton equivalent circuit degenerates into an ideal current-independent source having the current intensity equal to *Isc*;
6. A uni-port *N* that accepts, with respect to terminals A, B, neither a Thévenin equivalent circuit nor a Norton equivalent circuit is shown in Figure 5.4 (a). The *UAB – IAB characteristic* is defined by the equations

$$U_{AB} = 0 \quad I_{AB} = 0, \tag{5.6}$$

and therefore consists only of a point situated at the origin. It should be noted that the "virtual short circuit" that characterizes the input gate of an ideal op-amp, operating in the linear region, has this property. Such a gate is called a nullator.

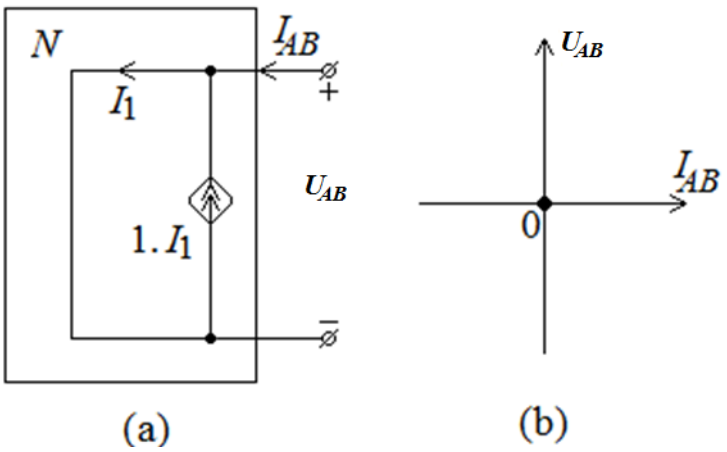


Fig. 5.4. A uni-port characterized by a point in the UAB plane – *IAB* – origin.

Since the function point characteristic for both the equivalent Thévenin and Norton ports consists of a straight line, it is clear that the nullifier does not have a single Thévenin or Norton equivalent port. Indeed, both 'single solubility conditions' are violated by this single port. Note that we can conduct *N* only with a voltage source of 0V or a current source of 0A.

From the above observations it can be seen that if the biport *N* is not controlled in current, then this circuit *does not* possess an equivalent Thévenin circuit. Dual, if *N* is not voltage controlled, then circuit *N does not* possess an equivalent Norton circuit. Thus, in applying the Thévenin or Norton theorem, one can ignore the verification of the "unique solvability condition" because it generally makes the verification of the determinant of the associated T-array matrix a difficult attempt. Instead, one can simply proceed to calculate *Req* or *Geq*. The impossibility of obtaining a single value for *Req* (respectively, *Geq*) could imply that the *circuit N* does not have an equivalent Thévenin circuit (respectively, Norton).

5.3. HYBRID EQUIVALENT CIRCUIT

Let be a linear resistive circuit containing resistors, independent voltage and/or current sources, and all four types of biport-controlled sources. We extract from the terminals (nodes) A and B of the circuit the resistor with the *resistance* R_{AB}, as in figure 5.10. The circuit to the left of nodes A, B in Figure 5.10 shall satisfy the conditions for equivalence of this circuit with the Thévenin equivalent circuit and with the Norton equivalent circuit, respectively. In order for a resistive linear circuit to be substituted, in relation to the terminals (nodes) A, B, with an equivalent Thévenin circuit, the voltage U_{AB} must exist and be unique for any value J of the current of an ideal source independent of current, when the resistor R_{AB} is replaced by such a source. Similarly, if the R_{AB} resistor in Figure 5.10 is replaced by an ideal voltage-independent source with t.e.m. E, the resistive linear circuit to the left of terminals A, B can be replaced by an equivalent Norton circuit if the current I_{AB} exists and is uniquely determined for any value E of the t.e.m. of the ideal source independent of voltage.

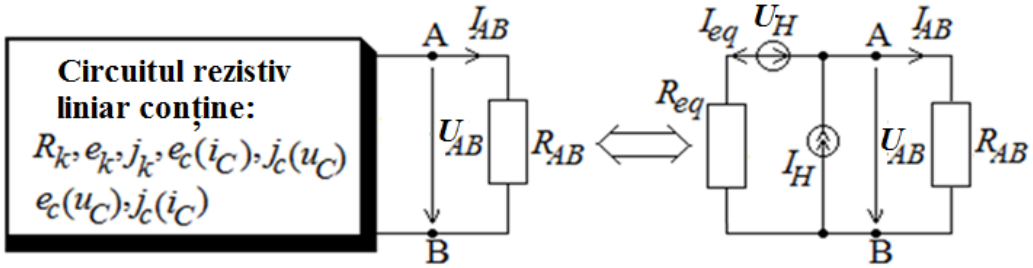


Figure 5.10. Hybrid equivalent circuit.

A hybrid equivalent circuit, or simply an H~ model, of a two-terminal network is a generalized version of the equivalent Thevenin or Norton circuit; for resistive circuits it consists of a U_H voltage source, an I_H current source, and an equivalent resistance, *Req*, which is identical to that in the Thévenin or Norton model (fig. 5.10). It seems that here one source, V_H or I_H, can be selected arbitrarily, and the other source is determined by equation (5.13).

We assume that the circuit to the left of the R_{AB} resistor in Figure 5.10 satisfies the conditions for the existence of the equivalent Thévenin and Norton circuits. According to the Thévenin theorem, the I_{AB} current has the expression:

$$I_{AB} = \frac{U_{Th}}{R_{AB} + R_{eq}}, \tag{5.13}$$

where is the voltage from terminals A, B of the active circuit, when $U_{Th} = U_{AB0} = U_{AB}|_{I_{AB}=0} I_{AB} = 0$, and the equivalent resistance of the passivated circuit with respect to terminals A, B ($R_{eq} = R_{AB0} = \frac{U_{Th}}{I_{ABsc}} I_{AB} = 0$). Applying the Kirchhoff theorems to the hybrid equivalent circuit in Figure 5.10, it follows:

$$\text{TIK-KCL: } I_{eq} = I_H - I_{AB} = I_H - \frac{U_{Th}}{R_{AB}+R_{eq}}, \tag{5.14}$$

$$\text{TIIK-KVL: } U_H = R_{AB}I_{AB} - R_{eq}I_{eq} = R_{AB}\frac{U_{Th}}{R_{AB}+R_{eq}} - R_{eq}\left(I_H - \frac{U_{Th}}{R_{AB}+R_{eq}}\right)$$

From equations (5.14) it follows:

$$I_H = I_{sc} - \frac{V_H}{R_{eq}} \text{ sau } U_H = U_{Th} - R_{eq} \cdot I_H, \tag{5.15}$$

where. It seems that here one source, $I_{sc} = I_{ABsc} = \frac{U_{Th}}{R_{eq}} U_H$ or I_H, can be selected arbitrarily, and the other source is determined by equations (5.15).

It should be noted that, like the Thévenin or Norton models, only two measurements are required here to obtain all the parameters of the H~ model. For example, for a selective value of the current I_H and two blood pressure measurements U_{Th} and the current $FLAX = Isc$ Equations $Req = U_{Th}/I_{sc}$ and (5.14) can be used to achieve strength Req and tension V_H for the model. Now, let's consider two circuits (networks) N_1 and N_2 connected via the j (Uj, I_j), as shown in Figure 5.11 [1]. There are two types of H~ models for the linear (linear) circuit (network) with two N terminals[1]. The H~ type 1 model is shown in Figure 5.10 (a). To find this model, first open the port where $Ij = 0$. If we refer to figure 5.10, a and considering equation (5.14) we get:

$$U_j = U_H + R_{eq}I_N = U_{Th} \tag{5.16}$$

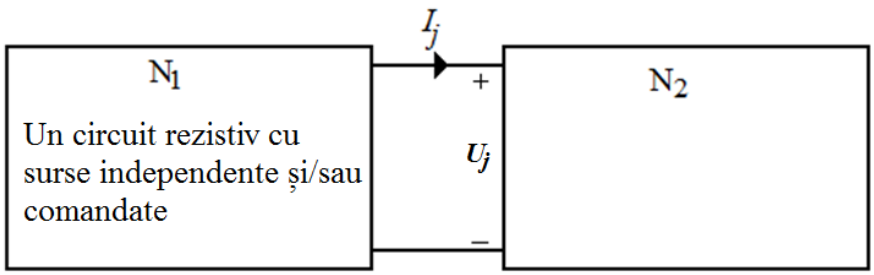


Figure 5.11. Two circuits (networks) N_1 and N_2 connected by a port j (Uj, I_j).

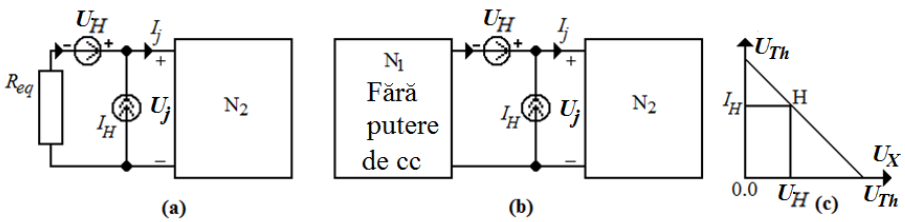


Figure 5.12. A hybrid equivalent circuit with two terminals for circuit N_1: a) Type 1 representation; b) Type 2 representation; c) The location on the characteristic curve of the port.

Equation (5.14) follows:

$$U_H = U_{Th} - R_{eq} I_N = U_{Th} - R_{eq} I_{sc}. \tag{5.17}$$

However, the sources of the H~ model of type 2 remain the same as those of type 1, but instead of calculating the equivalent resistance *Req*, we leave circuit N_1 remain unaltered, except that all its DC sources are removed, as shown in Figure 5.12(b). The term "*DC power output*" means that all independent DC sources are removed from the N $circuit_1$, including capacitor loads and coil currents. The H~ Type 2 model is useful in a number of applications, such as moving DC power sources from a circuit to its port-type terminals, without disturbing the internal structure (topology) of the circuit (network).

We must note that due to the fact that it has two sources instead of one, an H~ model represents an axis of freedom that acts as a tool in the dynamic modeling of a port. As shown in Figure 5.12(c), an H~- model covers a complete and continuous range of equivalent circuits for a two-terminal network. It is evident from equation (5.14) and figure 5.12 (c) that both the Thévenin and Norton models are two special cases of an H~ model.

A null hybrid equivalent circuit, called an H-model, is a special case of an H~model; where, the values of the voltage and current sources in the model are identical to the corresponding values of the voltage and current of the port. This means that the sources in an H model represent the bias situation of the corresponding port. For example, let's take the case in Figure 5.11, where the circuit (network) N_1 provides the voltage U_j and the current I_j to polarize the circuit (network) N_2. The two models for this example are shown in Figure 5.20 (a) and 5.20 (b). Note that Figures 5.20 (a) and 5.20 (b) are identical to Figures 5.12 (a) and 5.12 (b) unless the model sources represent the port values. We also note from Figure 5.20 that as a result of modeling H, another port is obtained, *k* (U_k, I_k), is created on N_1, where both U_K, as well as I_k I'm zero. Port *k* (U_k, I_k) is called *"null" port,* and the process of creating it is called "port cancellation", as will be discussed below.

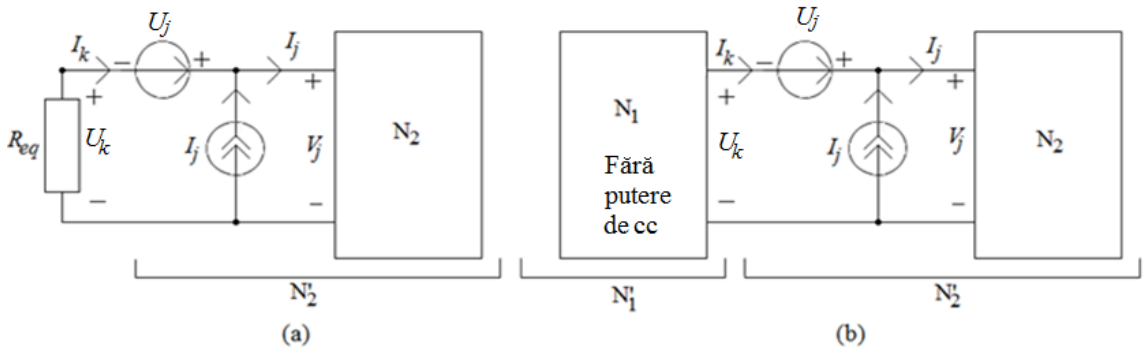


Figure 5.20. A model H for a terminal N_1 with two terminals; a) Type 1 representation; b) Type 2 representation, [1,2,15].

Theorem 1 introduces an important property of a model H that deals with the distribution of power in a network [1, 2, 15]. It adds an extra dimension to the analysis and segmentation of power in a network.

Theorem 1: Consider a circuit (a network) N_2 connected to another circuit (to another network) N_1 by a port *j* (V_j, I_j), as in figure 5.11. Replacing N_1 with Type 1 or Type 2 Model

H reduces energy consumption to zero, while energy consumption in N_2 remains unchanged, [13,14].

Proof: The H~ model in Fig. 5.12, (a) or 5.12, (b) is considered. Both sources, I_H and V_H, supply energy to the networks N_1 and N_2. The power delivered to the subcircuit N_2 is fixed and is calculated with the relation $P_2 = Uj*I_j$; while in the H~ model of type 1 the power consumed in the subcircuit N_1 (Fig. 3.10, a) is $P1 = Req(I_H - Ij)^2$. Therefore, the power P1 in the subcircuit N_1 becomes zero if $I_H = Ij$, which also results in the equality $V_H = V_j$. However, for model H type 2, we see from figure 5.16, (b) that subcircuit N'_1 has no direct current source from which to obtain power, in addition its port is also canceled. Therefore, all currents and voltages within the N'_1 subcircuit must be zero, resulting in zero energy consumption.

Port cancellation: A circuit (a network) N_2 connected to another circuit (to another network) N_1 through a port *j* (V_j, I_j) is considered, as shown in figure 5.21. One way to cancel port *j* is to increase the port on both sides (N_1 and N_2) with power sources. *Ij* and voltage sources *Uj* as shown in Figure 5.20. The result is the creation of another port *k* (*Uk*, I_k) which, by definition, is a null port, i.e. both *Ik* and *Uk* are zero, [1,2].

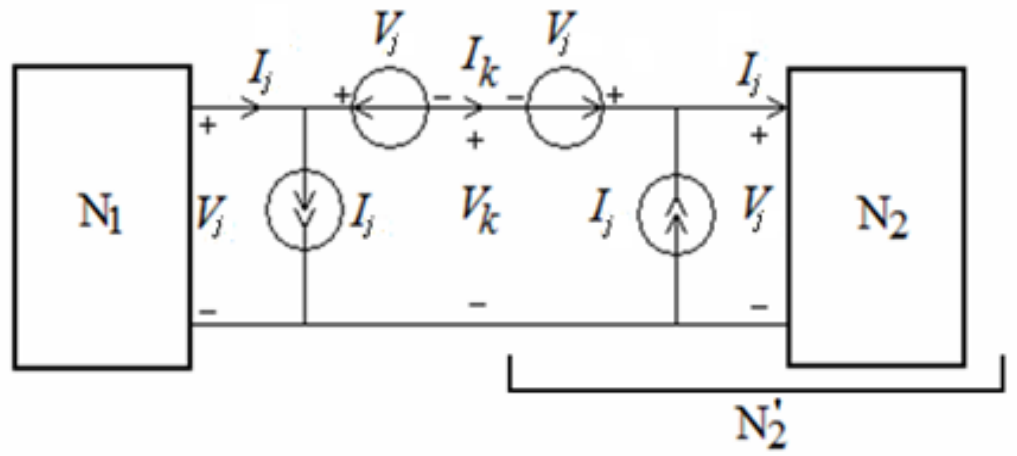


Figure 5.21. A simple port cancellation procedure, with no changes required N_1 or N_2, [1- 15].

However, there is an alternative method to create a null port when two circuits (networks) N_1 and N_2 are connected by a port (through a gate) *j* (*Uj*, I_j), shown in Figure 5.11. Here we can simply replace circuit N_1 with its Model H Type 1 or Type 2 and create the null port *k* (U_k, I_k) as shown in figure 5.18. As a result of the port cancellation procedure, shown in Figures 5.18 and 5.21, an extended network, N'_2, is created, which contains circuit N_2 and additionally contains the sources belonging to model H. Similarly, another network N'1 is created, on the left-hand side, when model H loses its sources. As we can see later, these extended networks are of great importance in the polarization of circuits. We must note that the characteristic curves of the gates (ports) *j* and *k* are identical, except for the displacements of *you* and *I*, the coordinate axes, from the origin to the point Q_j (*Uj*, I_j). This causes the operating point Q_j (*Uj*, *Ij*) to move to the origin, creating a new operating (operating) point $Q_k(0, 0)$ for port *k,* shown in Figure 5.22. This simply means that for any pair of networks, N_1 and N_2, connected by a port *j*, it is always possible to cancel the port and change the circuits N_1 and N_2 to circuits N'_1 and N'_2, where N'_1 and N'_2 are identical to N_1 and N_2, except for the *U*

and *I* coordinate axes that are at the point of operation (port operation). This is mentioned in Property 1.

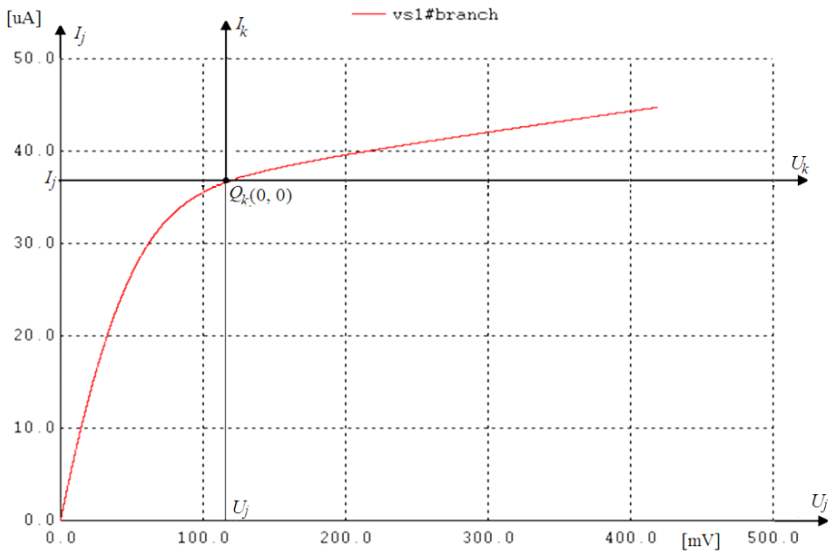


Figure 5.22. The I-U coordinate axis has moved from (0, 0) for port *j* to a new position, Q*j*(*Uj, Ij*), for port *k,* [15].

Property 1: Let two circuits (two networks) N_1 and N_2 be connected through a port *j*, as in Figure 5.11. If port j is null, then the characteristic curve I - *U* of the port, viewed in relation to either of the two networks, passes through the origin, and the origin is the point of operation of the port. If port *j* is not null, it is always possible to cancel the port to get the corresponding networks N'_1 and N'2 with a null port *k*, as shown in figure 5.18, [1,2].

Although the characteristic curve *I* – *U* of port *j* (associated with both networks) does not pass through the origin, the one that port *k* does (property 1). In addition, the Q point of the k port is located at the origin, as expected. It is noted that: i) the network N'*1*, on the left side, is still linear and ii) the new port *k* has a characteristic curve I – *U* passing through the origin, and the origin is also the point Q for the port. This simply means that the equivalent Thévenin circuit of N'*1*, looking from port *k*, must be a resistor with no source attached.

5.4. CONCLUSIONS

This chapter presents the necessary and sufficient conditions to be met by uniport (one-port) linear circuits in order to be replaced by the equivalent Thévenin, Norton and Hybrid circuits. These circuits are widely used in the analysis of analog circuits. It is simply proved, based on the superposition theorem, the theorems of Thévenin and Norton.

The main value of Thévenin's theorem, as well as Norton's theorem, is that it allows us to replace any part of a circuit that forms a linear resistive port, a linear circuit in a synusoidal regime, or a linear circuit in operation (but which is of no interest in a given situation) with only two circuit elements without affecting the solution of the rest of the circuit.

A new modeling technique, called H~, is introduced for single-port networks. It is shown that the H~ models are more dynamic compared to the equivalent Thevenin or Norton circuits and have the ability to describe the behavior of the port more precisely. Also, a special type of H~ model is introduced, called canceled H~ model, or simple H model; and many properties of H-modeling, including in-circuit power management, are investigated. A major property of H-modeling is the local polarization of transistors. Separates nonlinear components from the linear portion of the circuit for faster and more efficient circuit polarization. Here a designer can take advantage of H-modeling and polarization of individual transistors (or in combinations) without having to perform normal circuit polarization.

The Thévenin, Norton and hybrid equivalent circuits are used in the construction of Nullified Hybrid equivalent circuits. Thévenin, Norton, hybrid, and hybrid null equivalent circuits are used for the local polarization of analog circuits.

Another objective of this work is to introduce a guided design procedure for polarization. Our strategy separates the linear and nonlinear portions of an analog circuit and takes more control over the nonlinear portions. This separation of portions (components) within the circuit is achieved by introducing a new port modeling that cancels out the ports of nonlinear devices. This, in turn, leads to a new polarization technique for nonlinear components.

The use of equivalent Thévenin and Norton circuits in the simulation of nonlinear circuits with a small number of nonlinear circuit elements leads to a reduction in computation time and an increase in the accuracy of the results obtained.

6. CONCLUSIONS

The present paper has made significant contributions in the field of impedance adaptation, focusing specifically on the study and use of the Smith diagram in analog circuits and communication networks. By extending the classic functionalities of the Smith diagram, innovative methods have been developed that respond to the current challenges imposed by the complexity and dynamics of modern systems.

The main conclusions arising from this research are the following:

- The extension of the Smith diagram for non-linear and real circuits allowed a more faithful representation of the impedance behavior, which leads to a more precise and applicable adaptation under practical conditions.
- The development of computational algorithms integrated with the Smith diagram has facilitated automatic processes of design and adjustment of impedance adaptation, considerably reducing the time required and increasing the accuracy of the results, especially for applications with dynamic conditions and variations in circuit parameters.
- The introduction of a three-dimensional representation of the Smith diagram provided a new visual and analytical tool that allows the simultaneous analysis

of impedance variations over a wide range of frequencies, essential for the design of broadband circuits.

- The case study demonstrated the efficiency of the proposed methods, highlighting significant improvements in the reflection coefficient and the overall performance of the circuit, while validating the practical applicability of the developed solutions.
- The design of impedance matching networks is an essential process in radio frequency (RF) engineering, telecommunications, and microwave circuits, having a direct impact on the efficiency of power transmission between source and load.

In conclusion, this paper contributes to the advancement of knowledge in the field of impedance adaptation and provides a solid methodological and practical framework for the design and optimization of analog and communication systems. Future prospects include deepening adaptive methods and expanding applicability into new interdisciplinary fields, where impedance optimization plays a key role.

6.1. RESULTS OBTAINED

The results of this research are intended to contribute to the advancement of scientific knowledge and support the development of technologies in areas such as wireless communications, integrated signal processing systems, as well as electronic equipment with increased performance and energy efficiency requirements. Thus, the paper provides a solid platform for exploring and implementing innovative solutions in the field of impedance adaptation, supporting both academic and industrial efforts.

Through the numerous examples of the use of the Smith diagram in determining the circuit elements used in impedance adaptation, it confirms the outstanding efficiency of this diagram in the design of impedance adaptation schemes.

The necessary and sufficient conditions to be met by uniport linear circuits (one-port) to be replaced by the equivalent Thévenin, Norton and Hybrid circuits were presented. A new modeling technique, called H~, was introduced for single-port networks. The H~ models have been shown to be more dynamic compared to equivalent Thevenin or Norton circuits and have the ability to describe port behavior more accurately. Also, a special type of H~ model is introduced, called canceled H~ model, or simple H model; and many properties of H-modeling, including in-circuit power management, are investigated.

The use of equivalent Thévenin and Norton circuits in the simulation of nonlinear circuits with a small number of nonlinear circuit elements leads to a reduction in computation time and an increase in the accuracy of the results obtained.

6.2. ORIGINAL CONTRIBUTIONS

Original contributions are presented briefly, with each contribution specifying the original works in which it was published, by reference to the list in the next section.

In the context of the accelerated development of high-frequency communication systems, the design and optimization of RF (radio frequency) circuits and transmission lines is a major challenge for electronics engineers. Impedance matching is a field of research of utmost importance in electronic engineering and modern communication technologies, and has been extensively studied in the literature, in the context of optimizing the performance of analog circuits and communication systems.

The Smith diagram allows the design of adaptation networks that transform a given load impedance into the desired impedance (usually the conjugate of Z_S), and by using the Thévenin model, we can know exactly what impedance the load "sees" and we can use the Smith diagram to determine the necessary elements (e.g. a combination of L and C) to achieve the adaptation.

By combining these tools, the engineer has a complete framework at his disposal: the Thévenin/Norton theorem provides a simplified way to model the source or network prior to the task; the equivalent impedance extracted from these models is entered on the Smith diagram, allowing a graphical analysis of the fit; An adaptation network is designed to transform the load impedance into the complex conjugate of the source impedance, thus optimizing power transfer and minimizing reflections in the system.

By integrating Thevenin's and Norton's theorems with the Smith diagram, a highly efficient theoretical-practical approach for impedance matching is obtained. This methodology has direct applicability in the design of filters, amplifiers, RF networks and in all systems where signals must be transmitted efficiently between different stages of an electronic or communication chain.

The main original contributions made by the author in this doctoral thesis are:

- Some practical examples for using the Smith chart have been presented. Through the numerous examples of the use of the Smith diagram in determining the circuit elements used in impedance adaptation, it confirms the outstanding efficiency of this diagram in the design of impedance adaptation schemes
- The design and construction of the impedance adaptation are extensively presented by calculating with great accuracy the values of the parameters (coils and capacitors) used in the construction of the adaptation circuits.
- It has been shown that matching impedance over a wide frequency band will generally require complex, filter-like structures with many components, except in the mundane case of constant source and load resistances, when a transformer can be used.
- A maximum power point tracker used to extract maximum power from a solar panel and efficiently transfer it to batteries, the power grid, or other loads was showcased.

- The necessary and sufficient conditions to be met by uni-port linear circuits (one-port) are set out in order to be replaced by the equivalent Thévenin, Norton and Hybrid circuits. These circuits are widely used in the analysis of analog circuits. The Thévenin and Norton theorems are simply proved on the basis of the superposition theorem.
- It introduces a new modeling technique, called H~, for single-port networks. It is shown that the H~ models are more dynamic compared to the equivalent Thevenin or Norton circuits and have the ability to describe the behavior of the port more precisely. Also, a special type of H~ model is introduced, called canceled H~ model, or simple H model; and many properties of H-modeling, including in-circuit power management, are investigated. A major property of H-modeling is the local polarization of transistors. Separates nonlinear components from the linear portion of the circuit for faster and more efficient circuit polarization. Here a designer can take advantage of H-modeling and polarization of individual transistors (or in combinations) without having to perform normal circuit polarization.
- Development of a guided design procedure for polarization. The elaborate strategy separates the linear and nonlinear portions of an analog circuit and takes more control over the nonlinear portions. This separation of portions (components) within the circuit is achieved by introducing a new port modeling that cancels out the ports of nonlinear devices. This, in turn, leads to a new polarization technique for nonlinear components.
- It is shown how the equivalent circuits Thévenin, Norton and Hybrid can be used, with great success, in the simulation of nonlinear circuits with a small number of nonlinear circuit elements. This procedure leads to the reduction of the calculation time and to the increase of the accuracy of the results obtained.

6.3. LIST OF PUBLISHED WORKS

This numbered list includes the works published during the doctoral internship, with topics from the topic of the doctoral thesis, of which the doctoral student is the author or co-author. The works can also be found in the bibliography of the thesis.

1. Mihaela Grib, Ionela Vlad, Ovidiu Dorin Lăudatu, Alexandru Radu Grib, Mihai Iordache, „ Hybrid equivalent circuit generation", Carpathian Journal of Electrical Engineering Volume 15, Number 1, 2021
2. Mihaela Grib, Mihai Iordache, Alexandru Grib, Horatiu Popescu, Ovidiu Laudatu, Marius Staniloiu, „The use of Thévenin, Norton and hybrid equivalent circuits in the analysis and polarization of nonlinear analog circuits", 12th International Conference and Exposition on Electrical and Power Engineering, Iasi Romania, Oct 2022, https://doi.org/10.1109/EPE56121.2022.9959871
3. Zoltan Erdei, Cristian Barz, Alexandru Grib, Mihaela Grib, „Study of a Residual Current Device behavior for Half-Waves and Break Times Corelated with the Phase Angle of Tripping Currents", 2023 10th International Conference on Modern Power Systems (MPS) https://doi.org/10.1109/MPS58874.2023.10187564
4. Dragos Niculae, Mihai Iordache, Lavinia Bobaru, Marilena Stanculescu, Mihaela Grib, Ovidiu Laudatu, „Adapting the Impedance of The Signal Transmission Lines", 10th International Conference on Modern Power Systems (MPS), June 2023 https://doi.org/10.1109/MPS58874.2023.10187513
5. Mihaela Popescu, Marilena Stanculescu, Mihai Iordache, Lavinia Bobaru, Dragos Niculae, Mihaela Grib, „Computing the Parameters of the Impedance Circuit Matching", International Conference on Applied and Theoretical Electricity Craiova, Octombrie 2024, https://doi.org/10.1109/ICATE62934.2024.10748638
6. Mihaela Grib, Ioana-Gabriela Sirbu, Lucian Mandache, Marilena Stanculescu, Mihai Iordache, Lavinia Bobaru, and Dragos Niculae, "Generation Algorithms for Thévenin and Norton Equivalent Circuits", ENERGIES, Vol. 18, Issue6, DOI 10.3390/en18061344, Published MAR 10 2025, **WOS:001453861100001,** pp. 1 – 18.
7. Mihaela GRIB, Maria Lavinia BOBARU, Mihai IORDACHE, Marilena STĂNCULESCU, Georgiana REZMERIȚĂ, Mihai ROTARU, „The Use of Smith Diagram for Impedance Matching", Proceedings of the The 10th International Conference on Modern Power Systems 21 – 23 June 2023 (MPS 2023), Date Added to IEEE Xplore: 27 July 2023, ISBN Information:Electronic ISBN:979-8-3503-2682-6, Print on Demand(PoD) ISBN: 979-8-3503-2683-3, DOI: 10.1109/MPS58874.2023.10187467, Publisher: IEEE.
8. Mihaela GRIB, Mihai IORDACHE, Dragos NICOLAE, Marilena STĂNCULESCU, Maria Lavinia BOBARU, Alexandru GRIB, „Using the Smith diagram in the analysis of high-frequency analog circuits", Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.,Vol.70, 3, pp. 361–366, Bucarest, 2025, https://doi.org/10.59277/RRST-EE.2025.3.13

6.4. PROSPECTS FOR FURTHER DEVELOPMENT

The results of this research are intended to contribute to the advancement of scientific knowledge and support the development of technologies in fields such as wireless communications, integrated signal processing systems, and electronic equipment with increased performance and energy efficiency requirements.

Thus, the paper provides a solid platform for exploring and implementing innovative solutions in the field of impedance matching, supporting both academic and industrial efforts.

The Smith chart remains a fundamental tool for impedance matching in analog circuits, especially in RF circuit design. Its usefulness is notable not only for professionals in the field but also for radio amateurs who increasingly use modern tools for circuit or antenna analysis.

Future perspectives include deepening adaptive methods and expanding applicability to new interdisciplinary fields where impedance optimization plays a key role.

Through this research, it has been demonstrated that the Smith chart, combined with a solid understanding of impedance matching principles and equivalent models—such as the extended use of Thévenin, Norton, and Hybrid equivalent circuits in multiport electrical circuit analysis—constitutes a robust framework for efficient RF circuit design.

The results obtained can be extended to modern applications such as wireless communications, radar systems, and sensor networks, contributing to their improved performance and reliability.

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